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On the capacity of multi-packet reception enabled multi-channel multi-interface wireless networks

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ABSTRACT

Multi-packet reception (MPR) offers increased number of concurrent transmissions, which improves the capacity of wireless Ad hoc networks. By equipping each node with multiple wireless interfaces, the transmissions can be separated into different channels and the throughput can be increased. Combining these two technologies, this work studies the capacity of wireless networks wherein each node is equipped with multiple interfaces and each interface can decode at most k concurrent transmissions within its receiving range.

1. Introduction

According to the work of Gupta and Kumar [1], the capacity of wireless networks scales as $\Theta(W\sqrt{n})$ bit-m/s in arbitrary networks while scales as $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/s in random networks where W is the total bandwidth and n is the number of nodes in the network. Zhou et al. [2] studied the scaling laws for infrastructured wireless networks which encounter similar scalability problems. The per-node throughput degradation is due to the limited number of concurrent transmissions in the network.

Since a wireless interface usually can utilize multiple channels (e.g., IEEE 802.11b can employ 11 channels), this provides the potential to separate transmissions into different channels which will increase the number of concurrent transmissions in the network. On the other hand, as the successive interference cancellation (*SIC*) circuits with simple implementation and low complexity have been introduced, multi-packet reception (*MPR*) becomes a reality, which could increase the number of concurrent transmissions in the network. Assuming that all the transmissions within the receiving range could be decoded, past researches on the capacity of MPR-based wireless networks derived their results. However, as the number of simultaneous transmissions increases, the receiver cannot decode all of them. The explanation is shown below. As shown in [12], the main idea of *SIC* is to cancel each received signal one by one in the decreasing order of the signal strength and the signal cancellation process delay is restricted by the speed of performing Walsh–Hadamard Transform (*WHT*), so the possible number of cancellations is limited, which leads to the limited number of decoded transmissions.

In this paper, we utilize both multi-channel multi-interface and MPR ability to alleviate the traffic congestion of wireless Ad hoc networks. Using a more practical interference model compared with [8,9], this paper studies the capacity of 2-*D* MPR-based wireless networks, which are facilitated with multi-channel multi-interface. We assume

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that a wireless interface could decode at most $k, k \geqslant 1$ concurrent transmissions within its receiving range and each node could use m interfaces, each of which can work on one of the total c channels. We call such networks (m, c, k)-MPR wireless networks. For comparison, we call traditional networks 1-MPR wireless networks. The MPR ability, k, depends on the hardware implementation. We study how the capacity of 2-$D(m, c, k)$-MPR wireless networks scale with m, c, k and the number of nodes, n, in both arbitrary and random scenarios. We will show that in both arbitrary and random networks, the network capacity degrades as the ratio $\frac{c}{m}$ increases. However, for random networks, when $\frac{c}{m} = O(\log n)$ and $k = O\left(\sqrt{\frac{m \log n}{c}}\right)$, there is no capacity degradation as $\frac{c}{m}$ increases.

We organize the remainder of this paper as follows: we summarize the related work in Section 2. Section 3 describes the network model and main results. We prove the results for arbitrary networks in Section 4. In Section 5, we give the proof for the results of random networks. We discuss the results in Section 6. Section 7 concludes our paper.

2. Related work

The related work can be classified into two categories: multi-channel multi-interface category and multi-packet reception category.

For multi-channel multi-interface category, Bahl et al. [3] used only one interface to utilize multiple channels by channel switching. Vedantham et al. [4] also equipped each node with only one interface, however, without channel switching. They [4] issued a novel channel assignment method, component-based channel assignment, for wireless networks using multi-channel. This method achieves the same order of throughput as schemes using channel switching. Xing et al. [5] applied superimposed code for channel assignment in multi-channel multi-interface networks. This scheme supports both unicast and local broadcast effectively with low overall channel switching delay. Kyasanur and Vaidya [15] studied the capacity of multi-channel multi-interface wireless networks. They showed how the network capacity scales with the number of nodes, the ratio of channels to interfaces

For the multi-packet reception category, assuming that a node can concurrently send to and receive from many nodes when using FDMA and CDMA, Moraes et al. [6] studied the per source–destination throughput and the upper bound, lower bound of the Shannon capacity of each wireless link. Celik et al. [7] presented new backoff mechanisms for MPR-based wireless networks to deal with the unfairness and improve the network throughput. Assuming that all the transmissions within the receiving range of a receiver can be decoded, Garcia-Luna-Aceves et al. [8] have shown that 3-D random MPR-based wireless Ad hoc networks have a capacity gain of $\Theta(\log n)$ over traditional wireless Ad hoc networks. However, as the number of transmissions within the receiving range of a receiver increases, the receiver cannot decode all of them. To characterize the capacity of random MPR-based wireless networks in physical layer, Garcia-Luna-Aceves et al. [9] derived the upper bound and lower bound for the capacity of MPR-based wireless networks under physical model. Using a more realistic protocol model compared with [8], Wang and Garcia-Luna-Aceves [10] assumed that at most M simultaneous transmissions within the receiving range of a receiver can be decoded. They maximized the aggregated network throughput by formulating an optimization problem. In our previous work [11], we studied how the network capacity scales with the multi-packet reception ability.

According to the survey of the related work, we can see that this paper is the first work studying the capacity of 2-D MPR-based wireless networks using multi-channel multi-interface. Our contributions can be summarized as follows:

- To the best knowledge of us, this paper is the first work studying the capacity of such networks in both arbitrary and random scenarios.
- From the derived results, we get some valuable design implications for MPR enabled wireless networks using multi-channel multi-interface.

3. Network model and main results

3.1. Network model

In this paper, we suppose that there are c orthogonal channels, among which there are no interference. We also assume that each node is equipped with m identical wireless interfaces, $1 \leqslant m \leqslant c$, each of which can decode at most k transmissions within its receiving range. We call such multi-packet receiving ability k-MPR ability. We assume that by using all the c channels, the data rate is W bits/s. This data rate is divided equally among the c channels and therefore the data rate supported by each of the c channels is $\frac{W}{c}$ bits/s. Transmissions are slotted into synchronized slots of the same length τ. Packets are sent from source to destination in multi-hop.

Lemma 1. *When m equals c, the capacity of (m, c, k)-MPR wireless networks is equal to the capacity of (1,1,k)-MPR wireless networks.*

Proof. When m equals c, the c interfaces could work in parallel to minic the operation of the interface of each node in a $(1, 1, k)$-MPR wireless network. Consequently, the capacity of (m, m, k)-MPR wireless networks is at least the capacity of $(1, 1, k)$-MPR wireless networks. On the other hand, for $(1, 1, k)$-MPR wireless networks, there is only one interface and one channel with W bits/s for each node. Let us consider a simple extension of $(1, 1, k)$-MPR wireless networks. We divide the bandwidth of the single channel of $(1, 1, k)$-MPR wireless networks into m subchannels. For each node in the extended network, m interfaces are assigned for utilizing these m subchannels. Obviously, the network capacity of $(1, 1, k)$-MPR wireless networks is at least the capacity of such extended networks, which are actually (m, m, k)-MPR wireless networks. Hence, we get the lemma. $\square$

3.1.1. Arbitrary networks

In an arbitrary (m,c,k)-MPR wireless network, n nodes are arbitrarily located in a disk of unit area in the plane. Each node has an arbitrarily chosen destination to which it could send traffic at an arbitrary rate. Each node can choose arbitrary power level for each transmission. Under these assumptions, we give the protocol interference model for arbitrary (m,c,k)-MPR wireless networks as follow.

Suppose k nodes, $\{X_{ip}|1 \leqslant p \leqslant k\}$, transmit to node X_j simultaneously. These transmissions are successfully decoded by node X_j if

$$|X_q - X_j| \geqslant \max_{1\leqslant p\leqslant k}(1+\Delta)|X_{ip} - X_j|$$

for every other node X_q simultaneously transmitting over the same channel. Similar as [1], the quantity $\Delta > 0$ models a guard zone that prevents a neighboring node from transmitting on the same channel at the same time.

3.1.2. Random networks

In a random (m,c,k)-MPR wireless network, n nodes are randomly located, i.e., independently and uniformly distributed, on the surface of a torus of unit area. Each node has one flow to a randomly chosen destination to which it wishes to send at $\lambda(n)$ bits/s. Each node employs the same receiving range $r(n)$ for each reception. Under these assumptions, we give the protocol interference model for random (m,c,k)-MPR wireless networks as follow.

Suppose k nodes, $\{X_{ip}|1 \leqslant p \leqslant k\}$, transmit to node X_j simultaneously. These transmissions are successfully decoded by node X_j if

(1) The distance between each of the k nodes, X_{ip} and X_j is no more than $r(n)$, i.e.,

$$\max_{1\leqslant p\leqslant k}|X_{ip} - X_j| \leqslant r(n)$$

and

(2) For every other node X_q simultaneously transmitting over the same channel

$$|X_q - X_j| \geqslant (1+\Delta)r(n).$$

3.2. Main results

The results of this paper are presented as follows.

3.2.1. Arbitrary networks

The network capacity of arbitrary networks is measured in terms of "bit-m/s" (used in [1]). The upper bound and lower bound for the capacity of arbitrary networks match exactly. We present the network capacity as follows:

(i). When $\frac{ck}{m} = O(n), \lambda n\bar{L} = \Theta\left(W\sqrt{\frac{mnk}{c}}\right)$ bit-m/s.
(ii). When $\frac{ck}{m} = \Omega(n), \lambda n\bar{L} = \Theta\left(W\frac{mn}{c}\right)$ bit-m/s.

We also illustrate the capacity of arbitrary networks in Fig. 1.

3.2.2. Random networks

The network capacity of random networks is measured in terms of "bits/s". The upper bound and lower bound for the network capacity are presented as follows:

(1) Upper bound

When $k = O\left(\frac{\log n}{\log\log n}\right)$, the upper bound depends on $\frac{c}{m}$.

(i). When $\frac{c}{m} = O\left(\frac{\log n}{k}\right),\ \lambda n\bar{L} = O\left(Wk\sqrt{\frac{n}{\log n}}\right)$ bits/s.
(ii). When $\frac{c}{m} = \Omega\left(\frac{\log n}{k}\right)$ and $\frac{c}{m} = O(nk\left(\frac{\log\log n}{\log n}\right)^2),\ \lambda n\bar{L} = O\left(W\sqrt{\frac{mnk}{c}}\right)$ bits/s.
(iii). When $\frac{c}{m} = \Omega\left(nk\left(\frac{\log\log n}{\log n}\right)^2\right),\ \lambda n\bar{L} = O\left(W\frac{mnk\log\log n}{c\log n}\right)$ bits/s.

When $k = \Omega\left(\frac{\log n}{\log\log n}\right)$ and $k = O(n)$, the results are as follows:

(i). When $\frac{c}{m}$ is $O(\log\log n)$, the network capacity is $O\left(W\sqrt{\frac{kn}{\log\log n}}\right)$ bits/s.
(ii). When $\frac{c}{m}$ is $O(\frac{n}{k})$, $\frac{c}{m}$ is $\Omega(\log\log n)$ and k is $O\left(\frac{n}{\log\log n}\right)$, the network capacity is $O\left(W\sqrt{\frac{mnk}{c}}\right)$ bits/s.
(iii). When $\frac{c}{m}$ is $\Omega(\frac{n}{k})$, the network capacity is $O(W\frac{mn}{c})$ bits/s.

When $k = \Omega(n)$, the upper bound for network capacity depends on $\frac{c}{m}$.

(i). When $\frac{c}{m} = O\left(\sqrt{\log\log n}\right),\ \lambda n\bar{L} = O\left(W\frac{n}{\sqrt{\log\log n}}\right)$ bits/s.
(ii). When $\frac{c}{m} = \Omega(\sqrt{\log\log n}),\ \lambda n\bar{L} = O(W\frac{mn}{c})$ bits/s.

We also illustrate the upper bound in Fig. 2.

(2) Lower bound
(i). When $\frac{c}{m} = O(\log n)$ and $k = O\left(\sqrt{\frac{m\log n}{c}}\right),\ \lambda n\bar{L} = \Omega\left(Wk\sqrt{\frac{n}{\log n}}\right)$ bits/s.
(ii). When $\frac{c}{m} = O(\log n)$ and $k = \Omega\left(\sqrt{\frac{m\log n}{c}}\right),\ \lambda n\bar{L} = \Omega(W\sqrt{\frac{mn}{c}})$ bits/s.
(iii). When $\frac{c}{m} = \Omega(\log n)$ and $\frac{c}{m} = O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right),\ \lambda n\bar{L} = \Omega(W\sqrt{\frac{mn}{c}})$ bits/s.
(iv). When $\frac{c}{m} = \Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right),\ \lambda n\bar{L} = \Omega\left(W\frac{mn\log\log n}{c\log n}\right)$ bits/s.

We also illustrate the lower bound in Fig. 3.

4. Capacity of arbitrary networks

Firstly, we derive the upper bound for the capacity of arbitrary (m,c,k)-MPR wireless networks in Section 4.1. To show that the upper bound is tight, in Section 4.2 we give a constructive lower bound, which matches the upper bound.

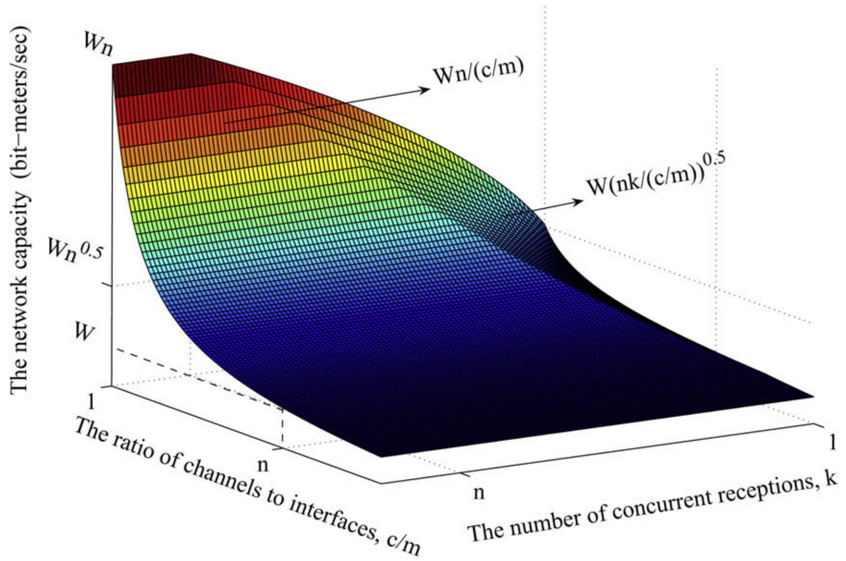

Fig. 1. The capacity of arbitrary (m, c, k)-MPR wireless networks (figure is not to scale).

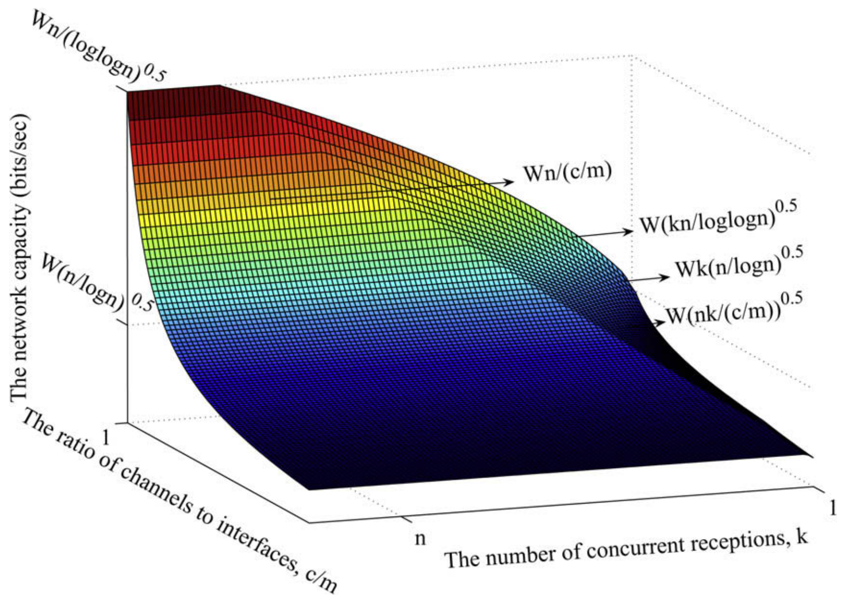

Fig. 2. The upper bound for the capacity of random (m, c, k)-MPR wireless networks (figure is not to scale).

4.1. Upper bound

In an arbitrary (m, c, k)-MPR wireless network, we suppose the whole network transports λnT over T seconds. The average distance between the source and destination of a bit is $\bar{L}$. Hence a capacity of $\lambda n\bar{L}$ is achieved. Under the network model described in Section 3.1, we have the following lemma.

Lemma 2. *Under the protocol model, the capacity of arbitrary (m, c, k)-MPR wireless networks is bounded as follows:*

(1). When $k = O(n)$, $\lambda n\bar{L} = O\left(W\sqrt{\frac{kmn}{c}}\right)$ bit-m/s.
(2). When $k = \Omega(n)$, $\lambda n\bar{L} = O\left(Wn\sqrt{\frac{m}{c}}\right)$ bit-m/s.

Proof. When $k = O(n)$, we use similar techniques used in [1] to prove the result. According to the network model we described above, we consider bit $b, 1 \leqslant b \leqslant \lambda nT$. Suppose that it moves from its source to its destination in a sequence of $h(b)$ hops, where the hth hop traverses a distance of r_b^h. Then we have

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} r_b^h \geqslant \lambda nT\bar{L}. \tag{1}$$

To utilize the k-MPR ability, we should group the nodes into $\left(\left\lfloor\frac{n}{k+1}\right\rfloor + 1\right)$ groups. Note that we are trying to upper bound the network capacity, hence such node grouping is

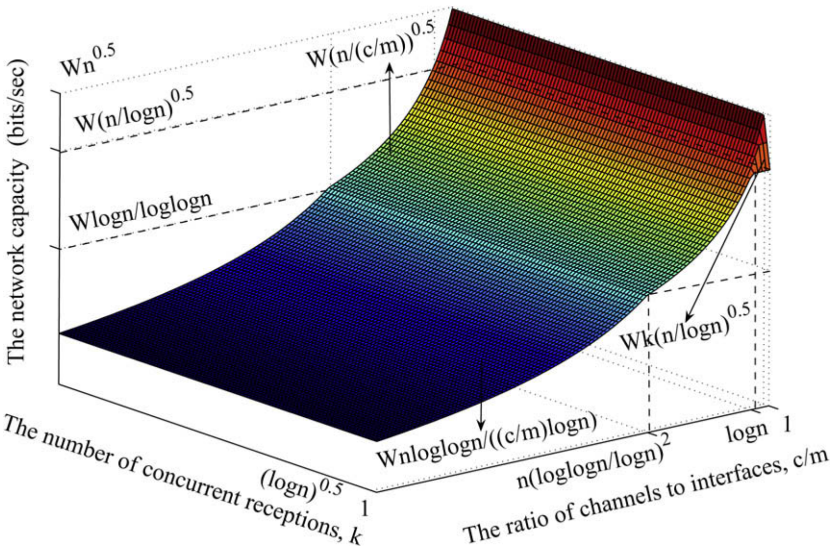

Fig. 3. The lower bound for the capacity of random (m,c,k)-MPR wireless networks (figure is not to scale).

only an ideal case and need not to be feasible. Each of these groups has k transmitters and one receiver. Because each node is equipped with m interfaces, the first $\left\lfloor\frac{n}{k+1}\right\rfloor$ groups totally contain at most $\left\lfloor\frac{n}{k+1}\right\rfloor mk$ transmission pairs and each of these groups contains at most mk transmission pairs. The last one group contains $m\left(n-\left\lfloor\frac{n}{k+1}\right\rfloor(k+1)-1\right)$ transmission pairs. Consequently, there are totally at most $m\left(n-\left\lfloor\frac{n}{k+1}\right\rfloor-1\right)$ transmissions in any time slot. Hence, we have

$$H=\sum_{b=1}^{\lambda nT}h(b)\leqslant\frac{WTm}{c}\left(n-\left\lfloor\frac{n}{k+1}\right\rfloor-1\right)\leqslant\frac{WTmn}{c}. \tag{2}$$

Recall the work of Gupta and Kumar [1]. Under the protocol model in [1], suppose node X_i transmits over the mth channel to a node X_j. Then this transmission is successfully received by node X_j if

$$|X_q-X_j|\ \geqslant\ (1+\Delta)|X_i-X_j| \tag{3}$$

for every other node X_q simultaneously transmitting over the same channel. The quantity Δ has the same meaning in their and our protocol model. According to their protocol model and triangle inequality, Ref. [1] concludes that disks of radius $\frac{\Delta}{2}$ times the lengths of hops centered at the receivers over the same channel in the same time slot are essentially disjoint. They have

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}1 \times (\text{The } h\text{th hop of bit } b \text{ is over channel } l \text{ in slot } s) \times\frac{\pi\Delta^2}{16}{r_b^h}^2\ \leqslant\ \frac{W\tau}{c}, \tag{4}$$

where in the parameters $\lambda,n,T,\Delta,b,h,h(b),r_b^h$ have the same meanings with our network model.

If we add k-MPR ability to this network, the number of concurrent transmissions within the disk of radius $\frac{\Delta}{2}r_b^h$ centered at a receiver is at most k. Hence in (m,c,k)-MPR wireless networks, (4) should be changed as follows:

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}1 \times (\text{The } h\text{th hop of bit } b \text{ is over channel } l \text{ in slot } s) \times\frac{\pi\Delta^2}{16k}{r_b^h}^2\ \leqslant\ \frac{W\tau}{c}. \tag{5}$$

Summing over the channels and the slots gives

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{\pi\Delta^2}{16k}(r_b^h)^2\ \leqslant\ WT.$$

This can be rewritten as

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}(r_b^h)^2\ \leqslant\ \frac{16kWT}{\pi\Delta^2 H}. \tag{6}$$

Note now that the quadratic function is convex. Hence

$$\left(\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}r_b^h\right)^2\ \leqslant\ \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}(r_b^h)^2. \tag{7}$$

Combining (6) and (7) yields

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}r_b^h\ \leqslant\ \sqrt{\frac{16kWTH}{\pi\Delta^2}}. \tag{8}$$

Now substituting (1) in (8) gives

$$\lambda nT\bar{L}\leqslant\sqrt{\frac{16kWTH}{\pi\Delta^2}}. \tag{9}$$

Substituting (2) in (9) yields the first result.

When $k = \Omega(n)$, because there are not enough transmitters to utilize the k-MPR ability, the network capacity cannot be further improved comparing with the case when $k = \Theta(n)$. Since the network capacity is $O(Wn\sqrt{\frac{m}{c}})$ bit-m/s when $k = \Theta(n)$, $\lambda n\bar{L} = O(Wn\sqrt{\frac{m}{c}})$ bit-m/s. □

Next, we bound the network capacity from another perspective.

Lemma 3. *The capacity of arbitrary (m, c, k)-MPR wireless networks, $\lambda n\bar{L}$, is $O(W\frac{mn}{c})$ bit-m/s.*

Proof. Because each node has m interfaces, there are totally mn interfaces in an arbitrary (m, c, k)-MPR wireless network. Because each interface can transmit at a rate of at most $\frac{W}{c}$ bits/s and the maximum distance a bit can travel in the network is O(1) meters, the network capacity is $O(W\frac{mn}{c})$ bit-m/s. □

According to Lemmas 2 and 3, the network capacity should satisfy these two constraints. We present the upper bound for the capacity of arbitrary (m, c, k)-MPR wireless networks as follows.

Theorem 1. *The upper bound on the capacity of arbitrary (m, c, k)-MPR wireless networks is as follows:*

(1). When $\frac{ck}{m} = O(n)$, $\lambda n\bar{L} = O\left(W\sqrt{\frac{mnk}{c}}\right)$ bit-m/s.
(2). When $\frac{ck}{m} = \Omega(n)$, $\lambda n\bar{L} = O\left(W\frac{mn}{c}\right)$ bit-m/s.

4.2. Lower bound

In this section, we achieve the upper bound to show that the upper bound is tight. For deriving the achieved lower bound, we present the following lemma for both arbitrary networks and random networks.

Lemma 4. *Assume $k_1 = O(k_2)$. C_1 is the network capacity of (m, c, k_1)-MPR wireless networks and C_2 is the network capacity of (m, c, k_2)-MPR wireless networks, then $C_1 = O(C_2)$.*

Proof. The (m, c, k_2)-MPR wireless networks can imitate (m, c, k_1)-MPR wireless networks by restricting k_2-MPR ability to k_1-MPR ability. Hence the capacity of (m, c, k_2)-MPR wireless networks is at least the capacity of (m, c, k_1)-MPR wireless networks. Hence we get the lemma. □

Next, we construct a $(1, \lfloor\frac{c}{m}\rfloor, k)$-MPR wireless network whose achieved capacity is a lower bound for the capacity of (m, c, k)-MPR wireless networks. The reason is presented as the following lemma, which has been proved in [15]. Although the lemma is presented in [15] for multi-channel multi-interface wireless networks without MPR ability, because (m, c, k)-MPR wireless network is a kind of multi-channel multi-interface wireless network, the lemma is also valid for (m, c, k)-MPR wireless networks.

Lemma 5. *Suppose m, c, c_a are positive integers such that $c_a = \frac{c}{m}$. Then the capacity of (m, c, k)-MPR wireless networks is at least the capacity of $(1, c_a, k)$-MPR wireless networks.*

Proof. Consider a (m, c, k)-MPR wireless network. We group the c channels into c_a groups (numbered from 1 to c_a), with m channels per group. Specifically, channel group $i, 1 \leqslant i \leqslant c_a$, contains all channels j such that $(i-1)m + 1 \leqslant j \leqslant im$. Suppose in a transmission group of a $(1, c_a, k)$-MPR wireless network, there are $q, 2 \leqslant q \leqslant k+1$, nodes, namely $X_j, 0 \leqslant j \leqslant q-1$, where X_0 is the receiver of this transmission group. Note that the nodes of this transmission group must work on the same channel, say, channel i. To simulate this behavior in (m, c, k)-MPR wireless networks, we can construct the same transmission groups as in $(1, c_a, k)$-MPR wireless networks. For the nodes of a group, we assign the m interfaces of each node to the m channels in the channel group i. In this fashion, the m interfaces of any node in the transmission group of the (m, c, k)-MPR networks are mapped to a channel group. The aggregated data rate of each channel group is $W\frac{m}{c} = \frac{W}{c_a}$. Therefore, a channel group in a (m, c, k)-MPR network can support the same data rate as a channel in the $(1, c_a, k)$-MPR network. This mapping allows the (m, c, k)-MPR network to minic the behavior of $(1, c_a, k)$-MPR networks. Hence, the capacity of (m, c, k)-MPR wireless networks is at least the capacity of $(1, c_a, k)$-MPR wireless networks. □

Lemma 6. *Suppose m and c are positive integers. Then the capacity of (m, c, k)-MPR wireless networks is at least $\frac{1}{2}$ the capacity of $(1, \lfloor\frac{c}{m}\rfloor, k)$-MPR wireless networks.*

Proof. When $\lfloor\frac{c}{m}\rfloor = \frac{c}{m}$, the result directly follows from the previous lemma. Otherwise, $m < c$. In this case, we use $c' = m\lfloor\frac{c}{m}\rfloor$ of the channels in the (m, c, k)-MPR network, and ignore all the other channels. Hence, a (m, c, k)-MPR network is degraded to a (m, c', k)-MPR, with a total data rate of $W' = W\frac{m}{c}\lfloor\frac{c}{m}\rfloor$. Using Lemma 5, the capacity of (m, c', k)-MPR wireless networks with total data rate of W' is at least the capacity of $(1, \lfloor\frac{c'}{m}\rfloor, k)$-MPR network with total data rate of W'. However, when $W' < W$, the capacity of (m, c', k)-MPR networks with total data rate W' is only a fraction, $\frac{W'}{W}$, of the capacity of $(1, \lfloor\frac{c}{m}\rfloor, k)$-MPR wireless networks with total data rate W (instead of W'). Since,

$$\frac{W'}{W} = \frac{m}{c}\left\lfloor\frac{c}{m}\right\rfloor = \frac{\lfloor\frac{c}{m}\rfloor}{\frac{c}{m}} \geqslant \frac{\lfloor\frac{c}{m}\rfloor}{\lfloor\frac{c}{m}\rfloor + 1}, \text{ since } \frac{c}{m} \leqslant \left\lfloor\frac{c}{m}\right\rfloor + 1$$
$$\geqslant \frac{1}{2}, \text{ since } \left\lfloor\frac{c}{m}\right\rfloor \geqslant 1.$$

Hence, the capacity of (m, c, k)-MPR wireless networks is at least $\frac{1}{2}$ the capacity of $(1, \lfloor\frac{c}{m}\rfloor, k)$-MPR wireless networks. Hence, the achieved capacity of $(1, \lfloor\frac{c}{m}\rfloor, k)$-MPR networks is a lower bound for the capacity of (m, c, k)-MPR networks. □

According to Lemma 6, we conclude that asymptotically, a (m, c, k)-MPR wireless network has at least the same order of capacity as a $(1, \lfloor\frac{c}{m}\rfloor, k)$-MPR wireless network.

We present the lower bound for the network capacity as follows:

Theorem 2. *The lower bound for the capacity of arbitrary (m, c, k)-MPR wireless networks is as follows:*

(1). When $\frac{ck}{m} = O(n)$, $\lambda n\bar{L} = \Omega\left(W\sqrt{\frac{mnk}{c}}\right)$ bit-m/s.
(2). When $\frac{ck}{m} = \Omega(n)$, $\lambda n\bar{L} = \Omega\left(W\frac{mn}{c}\right)$ bit-m/s.

Proof. To prove the theorem, we construct a $(1, c_a, k)$-MPR wireless network $\left(c_a = \lfloor\frac{c}{m}\rfloor\right)$, which achieves the capacity shown in the theorem. According to Lemma 6, the achieved capacity is a lower bound for the capacity of (m, c, k)-MPR wireless networks.

When $kc_a = O(n)$, we suppose that n is a multiple of $4(k+1)c_a$ (which does not impact the order) and define $r = \frac{1}{(1+2\Delta)\left(\sqrt{2\pi}+\sqrt{\frac{n}{2c_a(k+1)}}\right)}$. Since the deployment domain is a disk of unit area in the plane, we suppose that the center of the disk is located at the origin. For locations, $(j(1+2\Delta)r \pm \Delta r, i(1+2\Delta)r)$ and $(j(1+2\Delta)r, i(1+2\Delta)r \pm \Delta r)$, where $|j+i|$ is even, we place kc_a transmitters at each of them. We call such locations T locations and group the kc_a transmitters in each T location into c_a groups, each of which has k transmitters. For locations, $(j(1+2\Delta)r \pm \Delta r, i(1+2\Delta)r)$ and $(j(1+2\Delta)r, i(1+2\Delta)r \pm \Delta r)$, where $|j+i|$ is odd, we place c_a receivers at each of them. We call such locations R locations.

For each T location, the transmitters of the ith group transmit on channel i to the ith receiver in the nearest R location, which is at a distance r away where $1 \leqslant i \leqslant c_a$. It is easy to see that there is no interference among these transmissions under the protocol model. Using similar techniques of [1], we know that there are at least $\frac{nk}{k+1}$ concurrent transmissions within the domain. Restricting attention to only these transmissions, there are totally $\frac{nk}{k+1}$ simultaneous transmissions, each of which travels r meters and the interface of each node transmits at $\frac{W}{c_a}$ bits/s. This achieves the capacity of $\frac{1}{1+2\Delta}\frac{\sqrt{2c_a(k+1)}}{\sqrt{4\pi c_a(k+1)}+\sqrt{n}}\frac{nk}{k+1}\frac{W}{c_a}$ bit-m/s. Hence, when $kc_a = O(n)$, $\lambda n\bar{L} = \Omega\left(W\sqrt{\frac{kn}{c_a}}\right)$ bit-m/s.

When $kc_a = \Omega(n)$ and $k = O(n)$, $\frac{n}{k+1}$ of the nodes are selected as receivers, the other $\frac{nk}{k+1}$ nodes act as transmitters. We group the $\frac{nk}{k+1}$ transmitters into $\frac{n}{k+1}$ groups, each of which has k transmitters. Since the diameter of the unit disk is $\frac{2}{\sqrt{\pi}}$ meters, the distance between any transmitter and the receiver is no more than $\frac{2}{\sqrt{\pi}}$ meters and the transmission range can be bounded. We take a constant $r_0, 0 < r_0 < \frac{2}{\sqrt{\pi}}$ as common distance between any transmitter and its receiver. We place all the transmitters in one location, T_0 and place all the receivers in one location, R_0. The distance between R and T is r_0. The transmitters of the ith group transmit on channel i to the ith receiver where $1 \leqslant i \leqslant \frac{n}{k+1}$. There are $\frac{nk}{k+1}$ simultaneous transmissions in this network, each of which is at $\frac{W}{c_a}$ bits/s and each transmission travels r_0 meters. Hence, the achieved capacity is $\Theta\left(W\frac{n}{c_a}\right)$ bit-m/s. Hence, when $kc_a = \Omega(n), \lambda n\bar{L} = \Omega\left(W\frac{n}{c_a}\right)$ bit-m/s.

When $kc_a = \Omega(n)$ and $k = \Omega(n)$, according to Lemma 4, the achieved network capacity is at least the achieved capacity when $kc_a = \Omega(n)$ and $k = O(n)$. Hence, when $kc_a = \Omega(n)$ and $k = \Omega(n)$, $\lambda n\bar{L} = \Omega\left(W\frac{n}{c_a}\right)$ bit-m/s.

Recall that $c_a = \lfloor\frac{c}{m}\rfloor$. Because $\lfloor\frac{c}{m}\rfloor \leqslant \frac{c}{m}$, we have $c_a \leqslant \frac{c}{m}$. Substitute this inequation into the above results we get the theorem. □

5. Capacity of random networks

In a random k-MPR wireless network, we suppose that each node sends at $\lambda(n)$ bits/s to its destination. The highest value of $\lambda(n)$ which can be supported by every source–destination pair with high probability is defined as the per-node throughput of the network. The traffic between a source–destination pair is referred to as a "flow". Since there are totally n flows, the network capacity is defined to be $n\lambda(n)$. In Section 5.1, we give an upper bound for the network capacity. A constructive lower bound will be shown in Section 5.2.

5.1. Upper bound

The network capacity of random (m, c, k)-MPR wireless networks is limited by three constraints. The minimum of the three bounds is an upper bound of the network capacity.

5.1.1. Connectivity and k neighbors constraint

To bound the network capacity, we need to ensure that the random network is connected, so that every source–destination pair can successfully communicate. From another perspective, because we utilize k-MPR ability, we should ensure that each node has at least k neighbors over each of the c channels. According to the result of Wan and Yi [14], we can take the transmission range $r(n) \geqslant \sqrt{\frac{\log n + 2k\log\log n}{\pi n}}$ to satisfy this requirement. Note that the connectivity requirement is also satisfied by satisfying the k neighbors constraint. By satisfying these two constraints, in our previous work [11], we have given an upper bound for the capacity of random $(1,1,k)$-MPR wireless networks, which is applicable to (m, c, k)-MPR wireless networks as well.

(1). When $k = O\left(\frac{\log n}{\log\log n}\right)$, $n\lambda(n) = O\left(Wk\sqrt{\frac{n}{\log n}}\right)$ bits/s.
(2). When $k = \Omega\left(\frac{\log n}{\log\log n}\right)$ and $k = O(n)$, $n\lambda(n) = O\left(W\sqrt{\frac{kn}{\log\log n}}\right)$ bits/s.
(3). When $k = \Omega(n)$, $n\lambda(n) = O\left(\frac{Wn}{\sqrt{\log\log n}}\right)$ bits/s.

5.1.2. General constraint

A random (m, c, k)-MPR wireless network is a special case of arbitrary (m, c, k)-MPR wireless networks, hence the upper bound for the capacity of arbitrary (m, c, k)-MPR wireless networks is applicable to random networks. In a random network, the distance between any source–destination pair is $\Theta(1)$ meter. Hence, according to the upper bound for arbitrary networks, the capacity of random networks bounded as follows:

(1). When $\frac{ck}{m} = O(n)$, $n\lambda(n) = O\left(W\sqrt{\frac{mnk}{c}}\right)$ bits/s.
(2). When $\frac{ck}{m} = \Omega(n)$, $n\lambda(n) = O\left(W\frac{mn}{c}\right)$ bits/s.

5.1.3. Destination bottleneck constraint

The capacity of random (m, c, k)-MPR wireless networks is constrained by the data that can be received by a destination node. Note that each node picks a destination node randomly, so a node may be the destination of multiple flows. Let $D(n)$ be the maximum number of flows for which a node is the destination node. This number can be bounded using the following lemma. The lemma has been proved in [15].

Lemma 7. *The maximum number of flows for which a node is the destination node,* $D(n)$, *is* $\Theta\left(\frac{\log n}{\log\log n}\right)$ *with high probability.*

Consider a node X in a random (m, c, k)-MPR wireless network. X has at most $D(n)$ flows whose destination is X. Under the network model we described above, each channel supports a data rate of $\frac{W}{c}$ bits/s. Since we use k-MPR ability, the total data rate at which X can receive over m interfaces is $\frac{Wmk}{c}$. Because X should accommodate at least $D(n)$ flows, the data rate of the minimum rate flow is at most $\frac{Wmk}{cD(n)}$ bits/s. Therefore, by definition of $\lambda(n), \lambda(n) \leqslant \frac{Wmk}{cD(n)}$. Hence, the network capacity $n\lambda(n)$ is at most $O\left(\frac{Wmnk}{cD(n)}\right)$. According to Lemma 7, the network capacity is $O\left(\frac{Wmnk \log\log n}{c \log n}\right)$ bits/s.

By satisfying the three constraints above, the upper bound for the network capacity presented in Section 3.2 is got.

5.2. Lower bound

To establish a lower bound, we construct a routing scheme and a transmission scheduling scheme for any random $(1, c_a, k)$-MPR wireless network where $c_a = \lfloor \frac{c}{m} \rfloor$. According to Lemma 6, the achieved capacity is a lower bound for the capacity of random (m, c, k)-MPR wireless networks.

5.2.1. Cell construction

We divide the torus of unit area into square cells using the approach used in [13]. Each of such cells has an area of $a(n)$. We choose the receiving range of each wireless interface to be $\sqrt{8a(n)}$. With this range, all nodes in one cell can communicate with any node in its eight neighboring cells. The area of the cell, $a(n)$, should be chosen to satisfy multiple constraints, which will be described later. We set $a(n) = \min\left(\max\left(\frac{100\log n}{n}, \frac{kc_a}{n}\right), \left(\frac{1}{D(n)}\right)^2\right)$, where $D(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$ as described before. The first constraint is connectivity constraint. The cell size should be large enough, say, $a(n) \geqslant \frac{100\log n}{n}$, to guarantee network connectivity. The second constraint is used to guarantee that each node has at least kc_a neighbors, so each node can utilize the k-MPR ability on each of the c_a channels. The cell size should be large enough, say, $a(n) \geqslant \frac{kc_a}{n}$ to satisfy this constraint. The third constraint is to ensure that the network can at least accommodate the terminating flows of each node. The cell size should be chosen small enough, say, $a(n) \leqslant \left(\frac{\log\log n}{\log n}\right)^2$, to satisfy this constraint. We will discuss the third constraint in detail in the discussion on the routing scheme. For construction, we use the following lemma to bound the number of nodes in each cell. This lemma has been proved in [15].

Lemma 8. *If* $a(n) > \frac{50\log n}{n}$, *then each cell has* $\Theta(na(n))$ *nodes per cell, with high probability.*

By construction, we ensure that $a(n) \geqslant \frac{100\log n}{n}$ for large n by setting $a(n) = min\left(\max\left(\frac{100\log n}{n}, \frac{kc_a}{n}\right), \left(\frac{1}{D(n)}\right)^2\right)$. Consequently, with our choice of $a(n)$, Lemma 8 holds for suitably large n.

5.2.2. Routing scheme

The routing scheme includes two steps, cell assignment and node assignment respectively.

5.2.2.1. Cell assignment. In this step, we assign cells to serve each flow of the network. As shown in Fig. 4, packets of a flow are routed through the cells that lie along the straight line joining the source and the destination node. For each intersected cells of a flow (source–destination line), we choose a node to relay the traffic of this flow (we will describe the node assignment scheme later). We should consider a special case wherein the line exactly passes a grid point, say, Q, in Fig. 4. In this case, we require the cell on the right side of Q to serve this flow. (When you traveling from source node S to destination node D, the right side of Q is cell 2.) Hence, in Fig. 4, the packets of flow $S - D$ should be firstly relayed from cell 1 to cell 2 and then be relayed from cell 2 to cell 3.

5.2.2.2. Node assignment. For each flow served by a cell, we select a node from the cell to serve this flow. According to the cell assignment scheme, the next hop for any flow must be within one of the four neighboring cells: North, South, East, West. Each flow served by a cell can be classified by direction into one of four cardinal categories. A served flow whose next hop is within the northern neighboring cell is call a N flow served by this cell. Similarly, we can define S, E, W flows served by a cell. For example, as shown in Fig. 5, the flow from S_4 to D_4 is a N flow served by cell C.

The node assignment scheme has two steps. In step 1, the source node and destination node of a flow are assigned to serve the flow. In step 2, we assign nodes to serve those flows that pass through a cell. For each passing flow of a cell, the node, which has been assigned to the least

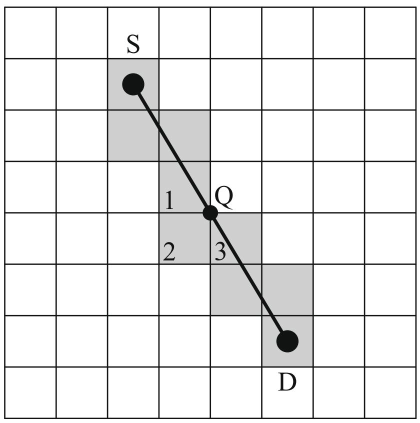

Fig. 4. Routing through cells in a random k-MPR wireless network.

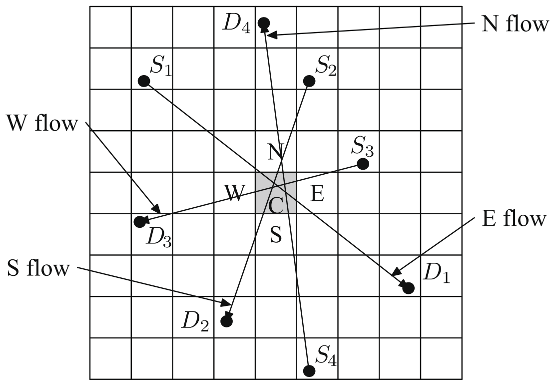

Fig. 5. The flows served by cell C are classified into four categories (E, S, W, N flows).

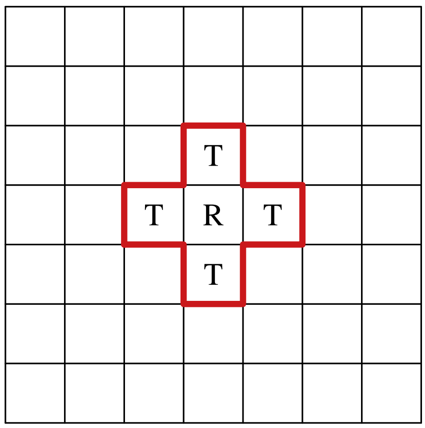

Fig. 6. Structure and its R-Cell and T-Cells in scheduling scheme.

number of flows of the same category so far, is assigned to serve the flow. This step evenly distributes the flows of each category among the nodes of a cell. Hence, for each flow category, all nodes serve nearly the same number of flows. The node assigned to a flow will receive packets from the assigned node in the previous cell and send the received packets to the assigned node in the next cell.

We present the following lemma to bound the number of source–destination lines that pass through any cell. This lemma has already been proved in [13].

Lemma 9. *The number of source–destination lines passing through any cell (including lines originating and terminating in the cell) is* $O(n\sqrt{a(n)})$ *with high probability.*

According to Lemmas 8 and 9, we conclude that each node serves $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows.

After introducing the routing scheme, we discuss the third constraint for cell size. Because each node should at least accommodate the flows who have the node as the destination, we should ensure $\frac{1}{\sqrt{a(n)}} = \Omega(D(n))$. Hence, the cell size $a(n)$ should not exceed $\left(\frac{\log\log n}{\log n}\right)^2$.

5.2.3. Scheduling transmissions

The scheduling scheme aims to schedule each flow with equal opportunity (i.e., all flows in a local region are served with the same number of time slots) while satisfying the following constraints:

(1) A node cannot transmit and receive simultaneously, since each node has only one wireless interface.
(2) A node cannot transmit to more than one receivers simultaneously. The reason is the same as that of constraint (1).
(3) Any two simultaneous transmissions should not interfere with each other under the protocol model.
(4) In order to utilize the k-MPR ability, the transmissions for the same receiver should be concurrent.

For scheduling transmissions, we define a new scheduling unit "Structure". In Fig. 6, the five cells surrounded by the red line compose a structure. We call the centering cell receiving cell (R-Cell). Each of the four neighboring cells of a R-Cell is called transmitting cell (T-Cell). For calculating the achieved network capacity, we consider the transmission scheduling in one second. We divide one second into several big slots, "Structure Slots". Then we further divide each structure slot into smaller slots, "Flow Slots".

We build a schedule using a two-layer process. The first layer is "Structure Layer" and the second layer is "Flow Layer". On the structure layer, we schedule the structures with structure slots to avoid conflicts and interferences between any two structures. On the flow layer, we schedule the individual flows with flow slots. After illustrating the two layer scheduling scheme, we will prove that the scheduling scheme schedules each flow with equal opportunity while satisfying the four constraints. Hence, this scheme is feasible.

5.2.3.1. Structure layer. On the structure layer, we schedule the structures satisfying the following requirements

(1) In any structure slot, a cell cannot act both as a T-Cell and as a R-Cell.
(2) In any structure slot, there is only one R-Cell for each T-Cell.
(3) In any structure slot, the transmissions of a structure should not interfere with the transmissions of any other structure under the protocol model.

Based on the three requirements, we introduce a definition **i-conflict structure**, $1 \leqslant i \leqslant 3$. If structure A and B cannot be scheduled in same structure slot due to requirement (i), $1 \leqslant i \leqslant 3$, we say that structure A is a **i-conflict structure** of structure B, vice versa. And we say that structure A and B are i-conflict structures with each other.

Theorem 3. *Under the protocol model, there is a schedule such that in every v structure slots, each structure in the tessellation gets one structure slot such that the three requirements are satisfied where v depends only on Δ.*

Proof. To prove the theorem, we should satisfy each of the three requirements.

Firstly, to satisfy the requirement (1), we should ensure that the 1-conflict structures are scheduled in different structure slots. As shown in Fig. 7, cell 1 acts as a *R*-Cell in the red structure (the left structure) while it acts as a *T*-Cell in the green structure (the right structure). Hence, red and green structures are 1-conflict structures with each other. In our grid tessellation, the number of 1-conflict structures for a structure is a constant, say, c'_1 (in fact, it is 4).

Secondly, as shown in Fig. 8, cell 2 acts as a *T*-Cell in the red structure (the lower left structure) while it acts as a *T*-Cell in the blue structure (the upper right structure). Hence, the red and blue structures are 2-conflict structures with each other. In our grid tessellation, the number of 2-conflict structures for a structure is constant, say, c'_2 (in fact, it is 8).

Thirdly, we will satisfy the requirement (3). Let N_1 denote the number of interfering cells of each cell in a random 1-MPR network (the cell tessellation is the same with that we described above). Let N_2 denote the number of interfering cells for each cell in a random *k*-MPR network. Let N_3 denote the number of interfering structures of each structure in a random *k*-MPR network. Since only the *R*-Cell of a structure can be interfered by other cells, we have $N_3 \leqslant N_2$. Since the only difference between the protocol models of 1-MPR and *k*-MPR networks is k concurrent reception, we have $N_2 \leqslant N_1$. Hence, we have $N_3 \leqslant N_1$. Ref. [15] has shown that N_1 is upper bounded by a number, say, c'_3, which depends only on Δ. Hence the number of 3-conflict structures for a structure is less than c'_3.

Letting each structure denote a node, we build an interference graph. There is an edge between two nodes, if the corresponding two structures of the two nodes are *i*-conflict structures, $1 \leqslant i \leqslant 3$, with each other. From the analysis above, we conclude that the maximum vertex degree of this interference graph is at most $c'_1 + c'_2 + c'_3$. Let $c' = c'_1 + c'_2 + c'_3$. According to [16], the interference graph can be vertex-colored with at most $(c' + 1)$ colors. Letting $v = c' + 1$, we get the theorem. □

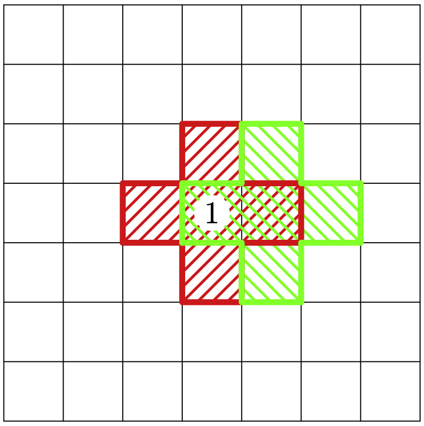

Fig. 7. 1-conflict structures.

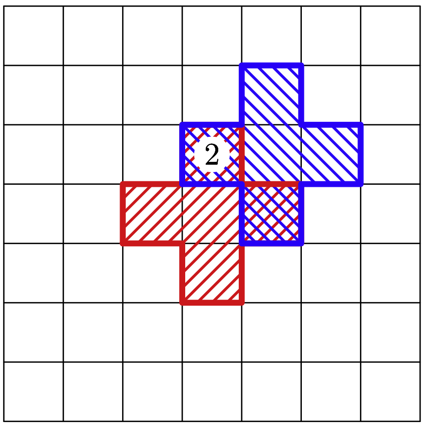

Fig. 8. 2-conflict structures.

5.2.3.2. Flow layer. On flow layer, we build a two-phase scheme to schedule each flow served by *R*-Cell using the *k*-MPR ability when a structure is scheduled for a structure slot. Recall that there is c_a channels in the network, so the transmissions can be separated into different channels to increase the network throughput. To achieve the scheduling objective, flow layer scheduling should ensure that each flow is served with equal number of flow slots in a structure slot. To be feasible, flow layer scheduling should satisfy the following requirements.

(1) In both phases, the number of concurrent receptions cannot exceed the number of transmitters.
(2) In second phase, the number of concurrent receptions cannot exceed the number of flows each node serves.

We will show that these requirements are necessary conditions for feasible flow layer scheduling later. Because of these requirements, we cannot always fully utilize the *k*-MPR ability and the feasible number of concurrent receptions cannot always be *k*. Hence, for flow layer scheduling, we define "g" as the feasible number of concurrent receptions where $g \leqslant k$.

5.2.3.3. First phase. In this phase, each flow served by *R* cell gets a flow slot to be transmitted from the *T*-Cell to the *R*-Cell. For each node in the *T*-Cells, each of the served flows whose next hops are in the *R*-Cell is transmitted for one flow slot. Since we use multi-packet reception, the number of receivers is less than the number of transmitters (ideally, the ratio between transmitters and receivers is *k*, when *k*-MPR ability is fully utilized). The other nodes in *R*-Cell do nothing in first phase. By construction, we arbitrarily select a portion (this portion, which is dependent on *g*, will be discussed later) of nodes from each cell and use them as receivers when the cell acts as a *R*-Cell in the first phase. We call these receivers of a *R*-Cell in first phase "Busy Nodes" and call all of the other nodes "Free Nodes". In the first phase, busy nodes receive transmissions on behalf of a *R*-Cell. Additionally, the flows served by free nodes but received by busy nodes in first phase should be distributed uniformly among the busy nodes for utilizing multi-packet reception in second phase.

5.2.3.4. Second phase. In this phase, each flow served by free nodes is assigned a flow slot to be transmitted from busy nodes to free nodes using the k-MPR ability. The transmitters in this phase are busy nodes.

Theorem 4. *Using the two-phase flow layer scheduling, for a scheduled structure, each flow served by the R-Cell can be assigned one effective flow slot to be transmitted from the serving node in the T-Cell to the serving node in the R-Cell.*

Proof. The flows served by the R-Cell can be classified into two categories. The first category includes all the flows served by the busy nodes of the R-Cell. The second category includes all the flows served by free nodes of R-Cell. For the first category, each of the flows is assigned only one flow slot in the first phase. Hence, each flow is transmitted from the serving node in the T-Cell to the serving node in the R-Cell for one effective flow slot. For the second category, each of the flows is assigned two flow slots (one in the first phase and one in the second phase). Since in the first assigned flow slot, the flow is transmitted from T-Cell to a temporary receiver, the number of effective assigned flow slots is only one (the one assigned in second phase). Hence, in flow layer scheduling, each of the flows served by the R-Cell gets one effective flow slot to be transmitted from the serving node in the T-Cell to the serving node in the R-Cell. $\square$

After introducing the two-phase process, we present the following theorem to show that the two requirements are necessary for feasible flow layer scheduling.

Theorem 5. *If flow layer scheduling with g concurrent receptions is feasible, then requirement (1) and (2) are satisfied.*

Proof. Firstly, we prove that requirement (1) is a necessary condition for feasible flow layer scheduling with g concurrent receptions. If requirement (1) is not satisfied, g is larger than the number of transmitters in the first or second phase. Hence, we have not enough transmitters to fully use g concurrent receptions and flow layer scheduling with g concurrent receptions is infeasible, which contradicts the assumption. Secondly, we prove that requirement (2) is a necessary condition for feasible flow layer scheduling with g concurrent receptions. If requirement (2) is not satisfied, g is larger than the number of flows each node serves in second phase. Hence, to make g concurrent receptions feasible, some or all of the flows served by free nodes are assigned more than one flow slot, because there are not enough flows for assigning each flow one flow slot to fully use g concurrent receptions. However, each of these flows is assigned only one flow slot in first phase and in each flow slot, a node sends at a data rate which is at most W bits/s, so the maximum number of bits of each flow to be transmitted in second phase is at most $W\cdot$ (*length of flow slot*). Hence, according to flow conservation, in second phase, in all the assigned flow slots for each flow (including the added flow slots), the total number of transmitted bits of the flow cannot exceed $W\cdot$ (*length of flow slot*). Consequently the added flow slots due to g concurrent receptions contribute nothing to the capacity. Hence, g concurrent receptions is infeasible, which contradicts the assumption. $\square$

We give the following theorem to justify the proposed two-layer scheduling scheme.

Theorem 6. *The proposed two-layer scheduling scheme is feasible and schedules each flow with equal opportunity.*

Proof. Because each cell can act either as a R-Cell or a T-Cell and according to the routing scheme all the flows served by a cell are from the four T-Cells, each flow in the network can be served. According to Theorem 4, we conclude that each flow is served with equal opportunity. Next, we prove that the scheme satisfies the four constraints to conclude that the scheme is feasible. Firstly, on the structure layer, a cell cannot act both as a R-Cell and act as a T-Cell in any structure slot and on the flow layer, a node cannot both transmit and receive in any flow slot, so constraint (1) is satisfied. Secondly, on the structure layer, there is only one R-Cell for each T-Cell in any structure slot and on the flow layer, a transmitter has only one receiver in any flow slot, so constraint (2) is satisfied. Thirdly, on the structure layer, the transmissions of a structure do not interfere with transmissions of any other structure under the protocol model in any structure slot and on the flow layer, at most k flows are scheduled in any flow slot, so there is no interference between any two transmissions under the protocol model and constraint (3) is satisfied. Fourthly, on the flow layer, g receptions of a receiver is scheduled in same flow slot, so the constraint (4) is satisfied. Hence, we get the theorem. $\square$

5.2.4. The achieved capacity lower bound

After introducing the whole scheduling scheme, we discuss the capacity lower bound for random (m,c,k)-MPR wireless networks. Recalling that each node serves $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows, hence, for requirement (2), we should ensure that the number of concurrent receptions is $O\left(\frac{1}{\sqrt{a(n)}}\right)$ in second phase.

(a). When $kc_a = O(\log n)$ and $k = O\left(\sqrt{\frac{\log n}{c_a}}\right)$, we have the following lemma for the capacity lower bound.

Lemma 10. *When $c_a = O(\log n)$ and $k = O\left(\sqrt{\frac{\log n}{c_a}}\right)$, we can use k-MPR ability on each of the c_a channels simultaneously.*

Proof. We can group all the transmitters in T-Cells into c_a groups. When $kc_a = O(\log n)$ and $k = O\left(\sqrt{\frac{\log n}{c_a}}\right), a(n) = \frac{100 \log n}{n}$. Suppose that we fully use the k-MPR ability. The number of transmitters on each channel in first phase is $\Theta(na(n)) = \Theta\left(\frac{\log n}{c_a}\right) = \Omega(k)$. Because $k = O\left(\sqrt{\frac{\log n}{c_a}}\right)$, the number of transmitters on each of the c_a channels in second phase is $\Theta\left(\frac{\log n}{kc_a}\right) = \Omega(k)$. Hence, requirement (1) is satisfied. Since $k = O\left(\sqrt{\frac{n}{\log n}}\right), k = O\left(\frac{1}{\sqrt{a(n)}}\right)$ and requirement (2) is satisfied. $\square$

Based on Lemma 10, we have the following lemma.

Lemma 11. *When $c_a = O(\log n)$ and $k = O\left(\sqrt{\frac{\log n}{c_a}}\right)$, $n\lambda n = \Omega\left(Wk\sqrt{\frac{n}{\log n}}\right)$ bits/s.*

Proof. In first phase, since the number of flows is $O(n\sqrt{a(n)})$ and we can use k-MPR ability on each of the c_a channels simultaneously, we need $O\left(\frac{n\sqrt{a(n)}}{kc_a}\right)$ flow slots to assign each flow one flow slot. In second phase, the number of flows needed to be redistributed from busy nodes to free nodes is $O((\frac{k-1}{k})n\sqrt{a(n)}) = O(n\sqrt{a(n)})$, so we also need $O\left(\frac{n\sqrt{a(n)}}{kc_a}\right)$ flow slots to assign each flow one flow slot. Consequently, we need $O\left(\frac{n\sqrt{a(n)}}{kc_a}\right)$ flow slots to schedule each flow with equal opportunity. We divide one second into v structure slots and divide each structure slot into $O\left(\frac{n\sqrt{a(n)}}{kc_a}\right)$ flow slots. Each flow slot has a length of $\frac{kc_a}{vn\sqrt{a(n)}}$ second. Since each interface can transmit at the rate of $\frac{W}{c_a}$ bits/s, in each flow slot, $\lambda(n) = \Omega\left(\frac{Wk}{n\sqrt{a(n)}}\right)$ bits can be transmitted. Since $a(n) = \frac{100\log n}{n}$, we get the lemma. □

(b). When $c_a = O(\log n)$ and $k = \Omega\left(\sqrt{\frac{\log n}{c_a}}\right)$, we present the following lemma for the capacity lower bound.

Lemma 12. *When $c_a = O(\log n)$ and $k = \Omega\left(\sqrt{\frac{\log n}{c_a}}\right)$, $n\lambda(n) = \Omega\left(W\sqrt{\frac{n}{c_a}}\right)$ bits/s.*

Proof. According to Lemma 4, the achieved network capacity when $c_a = O(\log n)$ and $k = \Omega\left(\sqrt{\frac{\log n}{c_a}}\right)$ is at least the achieved network capacity when $c_a = O(\log n)$ and $k = \Theta\left(\sqrt{\frac{\log n}{c_a}}\right)$. The achieved network capacity when $c_a = O(\log n)$ and $k = \Theta\left(\sqrt{\frac{\log n}{c_a}}\right)$ is $\Theta\left(W\sqrt{\frac{n}{c_a}}\right)$ bits/s, so we get the lemma. □

(c). When $c_a = \Omega(\log n)$ and $c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, we present the following lemmas for the capacity lower bound.

Lemma 13. *When $c_a = \Omega(\log n), c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^4\right)$ and $k = O\left(\sqrt{\frac{n}{c_a}}\right)$, we can partially use the k-MPR ability and the number of concurrent receptions is $\sqrt{k}$.*

Proof. When $c_a = \Omega(\log n), c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^4\right)$ and $k = O\left(\sqrt{\frac{n}{c_a}}\right), a(n) = \frac{kc_a}{n}$. Firstly, we prove that when $c_a = \Omega(\log n), c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^4\right)$ and $k = O\left(\sqrt{\frac{n}{c_a}}\right)$, we cannot fully use the k-MPR ability. If the number of concurrent receptions is k, in second phase the number of transmitters on each channel is $\frac{na(n)}{kc_a} = 1 = O(k)$, which contradicts the requirement (1). Secondly, we prove that we can partially use the k-MPR ability and the number of concurrent receptions is $\sqrt{k}$. With $\sqrt{k}$ concurrent receptions, the number of transmitters on each channel in first phase is $k = \Omega(\sqrt{k})$. The number of transmitters on each channel in second phase is $\sqrt{k}$. Hence, the requirement (1) is satisfied. Because $k = O\left(\sqrt{\frac{n}{c_a}}\right)$, $\sqrt{k} = O\left(\frac{1}{\sqrt{a(n)}}\right)$. Hence, the requirement (2) is satisfied. □

Based on Lemma 13, we present the following lemma.

Lemma 14. *When $c_a = \Omega(\log n)$, $c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^4\right)$ and $k = O\left(\sqrt{\frac{n}{c_a}}\right)$, $n\lambda(n) = \Omega\left(W\sqrt{\frac{n}{c_a}}\right)$ bits/s.*

Proof. When $c_a = \Omega(\log n)$, $c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^4\right)$ and $k = O\left(\sqrt{\frac{n}{c_a}}\right)$, the number of concurrent receptions is $\sqrt{k}$ and $a(n) = kc_a$. Substituting k with $\sqrt{k}$ and $a(n) = \frac{\log n}{n}$ with $a(n) = kc_a$, we use the similar technique used in proof for Lemma 11 to get the lemma. □

According to Lemma 4, when $c_a = \Omega(\log n)$, $c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^4\right)$ and $k = \Omega\left(\sqrt{\frac{n}{c_a}}\right), n\lambda(n) = \Omega\left(W\sqrt{\frac{n}{c_a}}\right)$ bits/s. Hence, we get the following lemma.

Lemma 15. *When $c_a = \Omega(\log n)$, $c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^4\right)$ $n\lambda(n) = \Omega\left(W\sqrt{\frac{n}{c_a}}\right)$ bits/s.*

Using similar techniques, we get the following lemma.

Lemma 16. *When $c_a = \Omega(n\left(\frac{\log\log n}{\log n}\right)^4)$ and $c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $n\lambda(n) = \Omega\left(W\sqrt{\frac{n}{c_a}}\right)$ bits/s.*

Combining Lemmas 15 and 16, we have the following lemma.

Lemma 17. *When $c_a = \Omega(\log n)$ and $c_a = O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $n\lambda(n) = \Omega\left(W\sqrt{\frac{n}{c_a}}\right)$ bits/s.*

(d). When $c_a = \Omega(n(\frac{\log\log n}{\log n})^2)$, we present the following lemma for capacity lower bound.

Lemma 18. *When $c_a = \Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $n\lambda(n) = \Omega\left(W\frac{n\log\log n}{c_a\log n}\right)$ bits/s.*

Proof. When $c_a = \Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, we use only $n\left(\frac{\log\log n}{\log n}\right)^2$ channels for transmission. On each of the $n\left(\frac{\log\log n}{\log n}\right)^2$ channels, the feasible number of concurrent receptions on each channel is only one. Hence, we need only the first phase of flow layer scheduling, because all the nodes in R-Cell can receive flows in first phase. In first phase, we need $O\left(\frac{n\sqrt{a(n)}}{1\cdot n\left(\frac{\log\log n}{\log n}\right)^2}\right)$ flow slots to assign each flow one flow slot. Using similar techniques used in the proof for Lemma 11, we get the lemma. □

6. Discussions

From the results for arbitrary (m, c, k)-MPR wireless networks, we conclude that when the ratio $\frac{ck}{m}$ is small enough, say, $O(n)$, the network capacity increases as the number of concurrent receptions, k, increases. Because, in this case, the m interfaces are sufficient to exploit the multiple wireless channel resources and the k-MPR ability can bring capacity gain to the network. However, when $\frac{ck}{m}$ is too large, there are not enough interfaces to fully exploit the wireless channel resources. Although the k-MPR ability can bring capacity gain, the capacity gain can be ignored compared with the capacity degradation due to limited number of interfaces.

Recall the results for random (m, c, k)-MPR wireless networks. When the number of concurrent receptions, k and the ratio $\frac{c}{m}$ are sufficiently small, the multiple wireless channel resources can be efficiently exploited by the m interfaces and there is no degradation for capacity gain compared with $(1, 1, k)$-MPR wireless networks. This result implies that we can equip the nodes with only a small number of wireless interfaces to exploit the wireless channel resources. However, when the ratio $\frac{c}{m}$ increases, the capacity gain brought by k-MPR ability can be ignored. Compared with arbitrary networks, when k increases, since we cannot intentionally plan the deployments of nodes to exploit k-MPR ability. The capacity of random networks cannot be further improved as k increases.

7. Conclusion

This work studied the capacity of wireless networks wherein each node is equipped with multiple interfaces, each of which can decode at most k transmissions within its receiving range. The results show that for both arbitrary and random scenarios, as the ratio of channels to interfaces increases, the capacity degrades and the capacity gain brought by the k-MPR ability can be ignored. However, when the ratio is small enough, there is no capacity degradation and the k-MPR ability dominates the order of the capacity gain.

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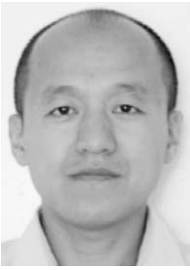

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