Capacity-aware Mechanisms for Service Overlay Design

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Abstract—We study mechanism designs for resource management in service overlay, where services are provided by strategic agents. Usually, resources in distributed systems are limited. However, the current mechanism design does not take the capacity of agents into consideration. Traditionally, the Vickrey-Clarke-Groves (VCG) mechanism has been the only method to design protocols so that each strategic agent will follow the protocols for its own interest to maximize its benefit. We show that the VCG mechanism is not truthful anymore when the capacity of agents is limited. Thus, we have designed, based on non-uniform prices, a new capacity-aware mechanism which subsidizes the service agents so that each agent maximizes its profit if it truthfully reports its cost. Mechanisms for two widely used pricing models are designed and evaluated.

I. INTRODUCTION

In the past decade, game theory models have been developed to understand network routing, Internet pricing, flow and congestion control, peer-to-peer systems, and many other issues [1]. A general service overlay consists of many strategic agents who will respond to incentives and act as service providers. The mechanism design borrows concepts from economics and game theory. It is capable of handling the strategic agents to produce a distributed incentive-compatible scheme. Therefore, maximizing benefit of each selfish agent leads to a global optimal system.

In general, most network service providers [2], [3] have limited resources to offer service to an increasingly expanding user group. When the load is about to reach its capacity limit, service provider may have a higher cost to guarantee the promised service to its users. In this paper, we will study the mechanism design in a capacity limited and climbing marginal cost setting. As shown later, truthful outcomes may exist for some representative price functions.

There have been many mechanisms proposed for distributed systems. In [4], the authors apply mechanism design to the overlay multicast problem to motivate each node towards a better multicast tree. In [5], the authors study game theory for distributed computing in respect of secret sharing and multiparty computation. More work can be found in [6] which summarizes current applications of distributed mechanism design. However, little work has been done to model and direct the behavior of selfish agents with limited capacities. Two related papers [7], [8] are listed. The authors of the former paper propose a progressive second price auction for sharing limited network bandwidth among multiple users. The authors of the latter paper implement a routing game, which assumes that the cost of shipping a unit of flow over any network link is proportional to the congestion level at the link. The common point is that they all focus their problems on the users’ side, i.e. how the users compete for the finite resources in the best economical way. In our work, we present another point of view from the service providers (abstracted as agents) to study the problem, i.e. how the agents behave to make higher profit for themselves, and whether there exists a strategy-proof mechanism which can obligle them to behave truthfully. To make it simple, one user with a total demand of some service is assumed to play with multiple capacity-constrained agents who provide it.

The remainder of this paper is organized as follows: in Section II, the concept of network mechanism design and our system model are illustrated. In Section III, we formulate some traditional methods and prove their inapplicability for the problem. In Section IV, we present our mechanism based on the system model. In Section V, conclusion and further work are given.

II. PRELIMINARIES AND MODELING

The mechanism design approach is briefly described as follows. Each agent has a type \( t_i \) known by agent \( i \) only. In fact, type \( t_i \) is the agent’s marginal cost. What agent \( i \) reveals is its strategy value \( a_i \) of its type (not necessarily the same as \( t_i \)) according to its strategy. In the system, there is a set of possible outcomes \( O(a^1, \ldots, a^n) \), depending on all \( n \) agents’ strategies. Each agent has a utility function in the function space \( U : O \rightarrow \mathbb{R} \), where \( u^i \in U \) that expresses its preference over these outcomes. Agent \( i \) chooses a strategy value \( a^i \) to maximize its utility \( u^i \). The utility function of an agent is known by itself. The desired system-wide goal is specified by a social choice function \( F : U \rightarrow O \) that maps each particular instantiation of agents, completely described by their utility functions, onto a particular outcome. A social choice function is strategy-proof if \( u^i(F(U)) \geq u^i(F(U^i \{ w \})) \), for all \( i \) and all \( w \in U \) (here \( U^i \{ w \} = \{ u^1, \ldots, u^{i-1}, w, u^{i+1}, \ldots, u^n \} \)). What this formula means is that if the agent reveals a specified strategy value of its type, the utilization will be maximized. In other words, with a strategy-proof mechanism, each agent has a dominant strategy (the strategy maximizing its utility regardless of other agents’ strategies), which always reveals the specified cost determined by the strategy. If \( F \) is strategy-proof and the dominant strategy for each agent is to reveal its...
real type (so called truthful), then no agent has an incentive to lie about its real cost, and the desired social goal can be achieved. The goal of mechanism design is to find a social function that is strategy-proof or truthful.

The traditional mechanism design does not take the capacity of agents into consideration. The famous Vickrey [9] - Clarke [10] - Groves [11] (VCG) mechanism has been proven to be the only truthful mechanism that has been successfully deployed to solve some network issues, the representative of which is the lowest-cost routing problem [12], [13]. In VCG, the type (cost, or more definitely marginal cost, the cost associated with one additional unit of production) of agent is fixed. However, an agent might have a capacity limit, and therefore, the marginal cost will increase with the load [8], [14], [15]. Here, two issues are to be addressed. First, the marginal cost will not be a constant any more. Instead, it will increase when the available capacity decreases. Second, although the real marginal cost may increase with the load, the agent could claim an even higher strategy value for higher utility. In this case, the mechanism is no longer truthful.

The problem described above is illustrated by the curves in Fig. 1. Assume that each of the two agents $A$ and $B$ has a capacity limit of 100 and 200 respectively. The marginal cost (MC) curves, average variable cost (AVC) curves, and average total cost (ATC, the average cost of production made up of variable cost combined with fixed cost) curves for the two agents are shown in Figs. 1(a) and (b). The fixed cost is 0.01 for agent $A$ and 0.005 for agent $B$. The supply curves are the same as the marginal cost curves. Fig. 1(c) shows the total supply (S) curve that is the horizontal sum of the supply curves of both the agents. A vertical demand (D) curve $F = 200$ is also shown. The intersection of the total supply curve and the demand curve determines the uniform price in the market. At this price ($p = 0.02$), the load distributed to agent $A$ and $B$ is $f_A = 50$ and $f_B = 150$, respectively. This load distribution leads to an efficient system. The payment is equal to the area of $ab f_A 0$ (ab $f_B 0$) and the total utility (profit) is equal to the area of abdc. When the total utility is larger than zero, the agent will be profitable. However, this analysis is only valid when the agents truthfully reveal their MC curves. In the world of oligopoly where a limited number of capacity-aware agents are present, the price may increase if the agents lie about MC curves. Is there an equilibrium price and how much is that price? Is there a dominant strategy for agents? Is there a mechanism that is strategy-proof? Does it lead to an efficient system? Since each agent will no longer reveal its real cost, a new mechanism needs to be investigated.

III. Uniform and Non-uniform Prices

Agents may apply various cost functions depending on the service to be provided in different overlay networks. They may behave quite diversely as follows:

- each agent claims its real cost as its strategy;
- each agent claims a higher strategy value than its real cost;
- each agent claims an infinite strategy value.

There will be a Nash Equilibrium for the first two cases, but not for the last. Also, the mechanism is not truthful in the last two cases. Our goal is to study whether there exists a Nash Equilibrium or even a truthful mechanism for a given cost function. Here, we present analysis for two representative price functions:

\[
\begin{align*}
    p_i &= F_1(q_i) = \frac{q_i}{r_i}, \\
    p_i &= F_2(q_i) = \frac{1}{x_i - q_i},
\end{align*}
\]

where price $p_i$ is the marginal cost varying as service quantity $q_i$, which reflects the load at agent $i$. Function $F_1$ is a linear supply function which does not have a hard capacity bound but lets the price increase proportionally with the flow [16]. This means the marginal cost of agent $i$ goes up with a fixed increment $1/r_i$, which represents the additional cost to take one more service quantity as the load. This function is simple and commonly presumed by many service overlay applications. In function $F_2$, as proposed in [8], $x_i$ is the capacity bound of agent $i$, and price $p_i$ increases rapidly with the load. When agent $i$'s load approaches the capacity bound, its marginal cost will accelerate up. This function is more practical since it emphasizes the impact of the remaining resources.

We will provide analysis for the profit of an agent and see how an agent behaves in order to reach its maximum profit.
The revenue of agent $i$ is defined as the product of its service quantity and price, $R_i = p_i \cdot q_i$, and the corresponding profit equals its revenue minus accumulated costs as the service load $q_i$ increases to its final market share $q^*$.

$$P_i = R_i - \int_0^{q^*} F(q_i) \partial q_i = p_i \cdot q^* - \int_0^{q^*} F(q_i) \partial q_i.$$ (3)

A. Maximum profit for $F_1()$

For $n$ agents with $q_i = F^{-1}_i(p_i)$, the total service demand $Q$ is formulated when the price $p$ is uniform in the market:

$$Q = q^1 + q^2 + \cdots + q^n = p \cdot \sum_{i=1}^{n} r_i.$$ 

For a particular agent $i$, who is interested in changing its $r_i$ to $r$ for a better profit, the total service demand is still fixed as

$$Q = (r + \sum_{j \neq i} r_j) \cdot p = (r + B_i) \cdot p,$$ (4)

where $B_i$, defined as $\sum_{j \neq i} r_j$, represents the sum of all other agents' claimed capacities.

It can be seen from Equ. 3 that in order to maximize profit $P_i$, revenue $R_i$ needs to be maximized first. Thus, for agent $i$ to claim $r$ as its rate, we analyze its $R_i$ first:

$$R_i = p_i \cdot q^i = p_i \cdot r^2 \cdot r = Q^2 \cdot \frac{r}{(r + B_i)^2},$$

$$\partial R_i = Q^2 \cdot \frac{B_i - x}{(r + B_i)^3}.$$ (5)

Agent $i$ can maximize its profit $P_i$ as below:

$$P_i = R_i - \int_0^{q^i} q_i \partial q_i \frac{r}{r_i} = R_i - \frac{1}{2 r_i} \cdot (q^i)^2.$$ Since the total service demand is still fixed, $Q = (r + \sum_{j \neq i} r_j) \cdot p = (r + B_i) \cdot q/r_i$.

Therefore,

$$P_i = R_i - Q^2 \cdot \frac{2 r_i}{(r + B_i)^2},$$

$$\partial P_i = \partial R_i - Q^2 \cdot \frac{2 r_i \cdot B_i}{(r + B_i)^3}.$$ (6)

By applying Equ. 5, $\partial P_i / \partial r = 0$, and we have

$$\frac{r_i \cdot B_i}{r_i + B_i} = \frac{r_i}{1 + \sum_{j \neq i} r_j}.$$ (7)

Hence, when $n = \infty$, $r$ approaching $r_i$, agent $i$ should retain its $r_i$ to maximize its profit. Otherwise, because $r_i / \sum_{j \neq i} r_j > 0$, agent $i$ will continuously decrease $r$, in hope of increasing its price, unboundedly.

B. Maximum profit for $F_2()$

Similarly, for $n$ agents with $q_i = F^{-1}_i(p_i)$, the total service demand $Q$ is formulated when the price $p$ is uniform in the market:

$$Q = q^1 + q^2 + \cdots + q^n = (\sum_{i=1}^{n} x_i) - \frac{1}{p_i} = (\sum_{i=1}^{n} x_i) - \frac{n}{p}.$$ (8)

For a particular agent $i$, who is interested in changing its $x_i$ to $x$ for a better profit, the total service demand is still fixed as

$$Q = (x + \sum_{j \neq i} x_j) - \frac{n}{p} = (x + B_i) - \frac{n}{p},$$ (9)

where $B_i$, defined as $\sum_{j \neq i} x_j$, represents the sum of all other agents' claimed capacities.

Again, for agent $i$ to claim $x$ as its capacity, we analyze its $R_i$ first:

$$R_i = p_i \cdot q^i = p_i \cdot x - 1 = \frac{nx}{x + B_i - Q} - 1,$$

$$\frac{\partial R_i}{\partial x} = \frac{n \cdot (B_i - Q)}{(x + B_i - Q)^2}.$$ (10)

Thus, for agent $i$ to maximize its profit $P_i$ by claiming $x$, we have

$$P_i = R_i - \int_0^{q^i} q_i \partial q_i \frac{r}{r_i} = R_i - \ln \left| \frac{x_i}{x_i - q^i} \right|.$$ Since the total service demand is still fixed, $Q = (x + \sum_{j \neq i} x_j) - n/p_i = (x + B_i) - n \cdot (x - q^i)$.

Therefore,

$$P_i = R_i - \ln \left| \frac{1 - nx}{n} \right| \cdot \frac{1}{x + B_i + Q} - 1,$$

$$\frac{\partial P_i}{\partial x} = \frac{n \cdot (B_i - Q)}{(x + B_i - Q)^2} - \frac{1}{(x + B_i - Q)} - x.$$ (11)

By applying Equ. 7, $\frac{\partial P_i}{\partial x} = 0$, and we have

$$x = -\frac{n + 2}{2} \cdot (B_i - Q) \pm \frac{n}{\sqrt{2(n + 3) \cdot (B_i - Q)^2 + 4x_i}}.$$ (12)

In this case, if $x_i + B_i - Q = nx_i (B_i - Q > 0)$, that is, the remaining capacity of the market equals $n$ times agent $i$’s capacity, and $x_i + B_i - Q = 0 (B_i - Q \leq 0)$, that is, no remaining capacity of the market is left, there may exist an equilibrium value $x = x_i$. Otherwise, $x$ will change continually to adjust the agent’s profit.

Next, we apply VCG mechanism to the same problem to check whether or not it can lead to an equilibrium among agents’ claims, or even a truthful result.
C. Maximum profit for $F_1()$ with VCG

According to the methodology of VCG, each agent following $F_1$ claims the type which will maximize its own profit. Then the price for that agent is calculated without taking its claim into consideration. To compute the price, let agent $i$’s type be $r = 0$, and the price in the market be $p = Q/B_i$. The payment to agent $i$ is calculated as the sum of the benefit and its claiming cost, where the benefit is the price drop resulting from agent $i$’s participation in the market. Then agent $i$’s profit is

$$P_i = \left( \frac{Q}{B_i} - \frac{Q}{r + B_i} \right) \cdot Q + \int_0^q \frac{q_i \partial q_i}{r} - \int_0^q \frac{q_i \partial q_i}{r_i} \right),$$

$$= \frac{Q^2}{B_i} - \frac{Q^2}{r + B_i} + \frac{Q^2 \cdot r}{2(r + B_i)^2} - \frac{Q^2 \cdot r^2}{2r_i \cdot (r + B_i)^2},$$

$$\frac{\partial P_i}{\partial r} = \frac{Q^2}{(r + B_i)^2} \cdot \left[ 1 + \frac{B_i - r}{2(r + B_i)} - \frac{r \cdot B_i}{r_i \cdot (r + B_i)} \right].$$

Let $\partial P_i/\partial r = 0$, and we have $r = 3B_i \cdot r_i/(2B_i - r_i)$. There is no equilibrium unless $r_i = 0$, so the VCG mechanism is not truthful in this scenario.

D. Maximum profit for $F_2()$ with VCG

Similarly, according to the methodology of VCG, each agent following $F_2$ claims the type which will maximize its own profit. Then the price for that agent is calculated without taking its claim into consideration. To compute the price, let agent $i$’s type be $x = 0$, and the price in the market be $p = n/(B_i - Q)$. The payment to agent $i$ is calculated as the sum of the benefit and its claiming cost, where the benefit is the price drop resulting from agent $i$’s participation in the market. Then agent $i$’s profit is

$$P_i = \left( \frac{n}{B_i - Q} - \frac{n}{x + B_i - Q} \right) \cdot Q + \int_0^q \frac{q_i \partial q_i}{x - q_i} - \int_0^q \frac{q_i \partial q_i}{x_i - q_i} \right),$$

$$= \frac{nQ}{B_i - Q} - \frac{nQ}{x + B_i - Q} + \ln |x| - \ln \left[ x + B_i - Q \right]_{n}$$

$$\quad - \ln |x_i| + \ln |x_i - x + \frac{x + B_i - Q}{n}|,$$

$$\frac{\partial P_i}{\partial x} = \frac{nQ}{(x + B_i - Q)^2} + \frac{1}{x} - \frac{1}{x + B_i - Q}$$

$$\quad - \frac{n - 1}{nx_i - (n - 1)x + B_i - Q}.$$

Let $\partial P_i/\partial x = 0$, and we find that it hardly has an equilibrium unless under very strict conditions. In this scenario, VCG mechanism is not truthful as long as the number of agents are finite. Therefore, a new mechanism is needed beyond the scope of traditional mechanism design.

IV. Capacity-aware Mechanism

A mechanism needs to be developed so that each agent maximizes its utility under the assumption of variable marginal costs. With a non-uniform price for each agent, the problem can be solved by this proposed capacity-aware mechanism. We first describe our mechanism and then prove its truthfulness.

CAM: Capacity-aware Mechanism

Step 1 The user claims a total demand $Q$ to $n$ agents.

Step 2 Each agent $i$ claims a type $t$ concerning how to maximize its profit, where type $t$ means cost rate $r$ in $F_1$ or capacity $x$ in $F_2$, respectively, and does not necessarily equal to its real type $t_i$.

Step 3 Then, $q_i^t$ and $p_i$ are calculated for agent $i$ as the price and quantity the user would like to assign to agent $i$.

- CAM-1: for $F_1$

$$q_i^t = Q \cdot \frac{r}{r + B_i},$$

$$p_i = \frac{Q}{2r_i} \cdot \left( \frac{r + B_i}{B_i} - \frac{B_i}{r + B_i} \right),$$

- CAM-2: for $F_2$

$$q_i^t = x - \frac{x + B_i - Q}{n},$$

$$p_i = \frac{n}{B_i - Q} \cdot \left[ \frac{n - 1}{n} \cdot x + B_i - Q \right].$$

where $B_i$ is the sum of all other agents’ claimed types.

Step 4 (only applies to $F_2$) If there exists any agent $i$ with $q_i^t \leq 0$, delete the agent who has the minimum claim of $x$ from the service-provider list and repeat Step 3 until $q_i^t > 0$ for every agent left in the service-provider list.

We assume that all the participants know the mechanism as well as the formulas to compute $q_i^t$ and $p_i$ before the trade. The user will comply with the rules to expect a truthful claim from the agents so as to prevent them from bidding up. Meanwhile, the agents will also comply with the rules because it will earn a profit by providing services at the price the user offers. We now go on to prove the following propositions.

**Proposition 1:** Capacity-aware Mechanism CAM-1 for cost function $q_i = F_1^{-1}(p_i) = r_i \cdot p_i$ is a dominant strategy.

**Proof.** From Equ. 3 and 8, the profit of agent $i$ can be computed as:

$$P_i = p_i \cdot q_i^t - \int_0^q \frac{q_i \partial q_i}{r_i} = p_i \cdot q_i^t - \frac{(q_i^t)^2}{2r_i}$$

$$= \frac{Q^2}{2B_i} - \frac{Q^2 \cdot B_i}{2r_i} \cdot \frac{r^2}{(r + B_i)^2},$$

$$\frac{\partial P_i}{\partial r} = \frac{Q^2 \cdot B_i}{(r + B_i)^3} - \frac{Q^2 \cdot (2r_i \cdot (r + B_i)^3)}{2r_i} \cdot \frac{Q^2 \cdot B_i}{(r + B_i)^3} \cdot (1 - \frac{r}{r_i}).$$
Hence let $\partial P_i/\partial r = 0$, and we have $r = r_i$. □

The agents will reveal their real types in order to gain their maximum profits. The user achieves this goal by offering a little bit higher price than the normal uniform price. Based on Eq. 4 and 8, the extra price paid to agent $i$ is

$$
\Delta p_i = \frac{Q}{2r} \left( \frac{r + B_i}{B_i} - \frac{B_i}{r + B_i} \right) - \frac{Q}{r + B_i} = \frac{Q}{2B_i} \cdot \frac{r}{r + B_i}.
$$

**Proposition 2:** Capacity-aware Mechanism CAM-2 for cost function $q_i = F^{-1}_2(p_i) = x_i - 1/p_i$ is a dominant strategy.

**Proof.** From Eqn. 3 and 9, the profit of agent $i$ can be computed as:

$$
P_i = p_i \cdot q_i - \int_0^{q_i} \frac{\partial x}{x_i - x} = p_i \cdot q_i - \ln |x_i| + \ln |x_i - q_i|$$

$$
= (n - 1) \ln \left( \frac{n - 1}{n} \cdot \frac{x + B_i - Q}{B_i - Q} \right) - \ln |x_i|$$

$$
+ \ln \left( \frac{1 - n}{n} \cdot \frac{x_i + B_i - Q}{n} \right),
$$

$$
\frac{\partial P_i}{\partial x} = \frac{n - 1}{n} \cdot \frac{1}{x + B_i - Q} - \frac{n}{n-1} \cdot \left( \frac{x_i + B_i - Q}{n} \right).$$

Hence let $\partial P_i/\partial x = 0$, and we have $x = x_i$.

Now we consider the situation that agent $i$ is allocated with zero service quantity due to its negative result of $q_i$ in step 3. Thereafter, agent $i$ attempts to overclaim its type $x$ instead of its real type $x_i$ in order to obtain a non-negative $q_i$. However, since agent $i$ is unaware of $B_i$, it cannot ensure whether or what type of changes of $x$ could lead to a better service allocation as well as an improved utility. Therefore, claiming its truthful type still remains as its dominate strategy. □

The agents will reveal their real types in order to gain maximum profits. The user achieves this goal by offering a little bit higher price than the normal uniform price. The extra unit price paid to agent $i$ is

$$
\Delta p_i = \frac{n}{x - B_i - Q} \cdot \ln \left( \frac{n - 1}{n} \cdot \frac{x + B_i - Q}{B_i - Q} \right) - \frac{n}{x + B_i - Q}.
$$

**V. CONCLUSION AND FURTHER WORK**

Proxy-based, peer-to-peer, and wireless overlay networks become more popular in recent years. One common feature of these networks is that they all have constrained capacities when providing services to comparatively large amount of customers. The question arises that how can they allocate their resources in the most profitable way. In this paper, we apply mechanism design and microeconomics to this specified problem. We have demonstrated that the VCG mechanism is not truthful in capacity-constrained settings. We have also proved that a uniform price in oligopoly situation can not reach equilibrium in our models. So we propose a new mechanism with non-uniform prices for distinct capacity-aware agents. This mechanism ensures a truthful outcome and leads to an efficient system.

Our further work is to study the generalization of our mechanisms for any cost functions with limited capacity. We are specially interested to know that for certain kind of functions, whether a capacity-aware mechanism exists or not.

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