

Quality Evaluation of Vehicle Navigation with Cyber Physical Systems

Yang Yang^{*}, Xu Li[†], Wei Shu[♦] and Min-You Wu^{*}

^{*}Department of Computer Science and Engineering, Shanghai Jiao Tong University, China

[†]Department of Computer Science and Engineering, State University of New York at Buffalo, USA

[♦]Electrical and Computer Engineering Department, The University of New Mexico, USA

Abstract—In this article, we focus on a typical application of Cyber Physical System (CPS), i.e., *vehicle route navigation*. Two fundamental problems have been examined: 1) How different are various vehicle routing algorithms? 2) How valuable is real-time traffic information or historical traffic information in helping vehicle routing? Different from most previous works based on simulations, we presented performance comparisons of four routing algorithms using real GPS sensory data from 4000 taxis. It has been shown that using real-time traffic information could substantially improve the quality of vehicle path routing. This can benefit an individual driver, because through our evaluation results on realistic vehicle traces we found that the paths selected by taxi drivers are not as good as expected. More importantly, utilizing real-time information could improve global transportation efficiency in terms of dispersing/managing traffic, which plays a key role in constructing an effective vehicular CPS.

Keywords—*Vehicle path routing, real-time traffic information; Vehicle Infrastructure Integration; Cyber Physical System; IntelliDrive*

I. INTRODUCTION

Recently, academic and industry community proposed the idea of communications and integration among vehicles and infrastructure within a transportation system, namely, *Vehicle Infrastructure Integration (VII)*. The objective of this idea is not only to *passively* monitor traffic but also to *actively* control and manage traffic, which is also called *Cyber Physical Systems (CPS)* in computer science domain.

In this paper, we focus on a typical CPS application, i.e., *vehicle route navigation* [1][2], in which vehicles are assumed to be equipped with on-board navigation system. Different from most previous works, which mainly discussed algorithm design, we investigate the advantage of utilizing real-time traffic information for routing. We want to answer two fundamental questions: 1) *How different are various vehicle routing algorithms?* 2) *How valuable is real-time traffic information or historical traffic information in helping vehicle routing?* Since large-scale real deployment leads to considerable cost, most of existing works relied on simulations or artificial traces, which have limitations to reflect realistic transportation system and driver behaviors. Fortunately, we are able to utilize real-time data from GPS sensors of 4,000 taxis in Shanghai [6][8]. We mapped these data onto a digital map to trace vehicles and to evaluate various routing algorithms. We found that both historical and real-time traffic information can improve the performance of vehicle routing. This paper provides a first-hand report about the worthiness of real-time traffic information for vehicle route navigation, which may serve as a guideline when deploying large-scale vehicular navigation systems. Some interesting insights have also been presented, e.g., we found that the paths selected by taxi drivers are not as

good as expected. It is worth noting that our results not only showed how individual driver can benefit from real-time traffic information, but also it demonstrated that utilizing real-time information could improve global transportation efficiency in terms of dispersing/managing traffic, which contributes to a key component of an effective vehicular CPS.

II. RELATED WORK

Finding shortest path is a classical problem in graph theory. Most of related problems can be finally transformed to a shortest path problem, which has two properties: static or dynamic, deterministic or stochastic [5][11]. For the static deterministic case, all data are known in advance and they are independent on time [12] and this is a classical shortest path problem [2]. For the static stochastic case, the input data is both random and time-independent. Eiger et al [3] described that when the utility function is linear or exponential, an efficient Dijkstra-type algorithm can be used to find the optimal path. For the dynamic stochastic case, the input data is also random and time-dependent. Miller-Hooks [9] denoted that a single path cannot provide an adequate solution for a given source-destination pair at a specific departure time, because the optimal path depends on intermediate information. For the dynamic deterministic case, all data are also known in advance and they are time-dependent. Kaufman and Smith [7] showed that the standard shortest path (such as the Dijkstra [2] algorithm) can be used as long as the network satisfies a deterministic consistency condition (refer as FIFO). Chabini's work [1] showed the optimal routing path equations and the corresponding proofs in single source with ADET algorithms and single destination with SDOT algorithm. By using time space network, Pallottino [10] presented a general "chronological" algorithmic paradigm, called Chrono-SPT, which can be used to get the optimal routing path in discrete-time dynamic networks.

Overall, those existing works are based on specific assumptions and simulations. Testing of various algorithms in the real traffic network is seldom to be found. We have built up a vehicular CPS, called the *Shanghai Urban Vehicular Network (SUVnet)* by collecting the GPS data from thousands of taxis. The SUVnet has been initially designed and developed by our research team at Shanghai Jiao Tong University. This platform has provided us with a foundation for real time information collection and dissemination, as reported in our previous work [6]. We have also investigated the traffic-monitoring performance obtained by SUVNet and showed that the estimations of traffic statuses are reasonably accurate [9]. Therefore, estimations of the traffic status have been used in this article for vehicle path routing.

III. VEHICLE ROUTING ALGORITHMS

We first introduce some terms and performance metrics used in this article. A traffic network $G(N, E)$ includes a set of nodes (intersections) and a set of edges (road sections). Con-

sider $n_S \in N$ as the source and $n_D \in N$ as the destination, path $p(S, D)$ is then a sequence of nodes:

$$p(S, D) = \{n_1, \dots, n_m \mid n_1 \equiv n_S, n_m \equiv n_D, m > 1, \\ e(n_k, n_{k+1}) \in E \text{ for } 1 \leq k < m\}$$

In general, there are several properties associated with $e(i, j)$, some of them are dependent of time t and the others are not.

Edge distance (ED), $d(e(i, j))$, time-independent. It is the physical length of road section from n_i to n_j .

Road design speed (RDS), $s_R(e(i, j))$, time-independent. Based on the road's physical capacity such as width, steepness in slope and curve, it is a speed on edge recommended for regular vehicles. Their values change substantially for different road sections but are time-independent. It is commonly used by current vehicle navigation system for vehicle routing.

Real-time speed (RTS), $s_R(t, e(i, j))$, time-dependent. It is the real-time average speed of vehicles traveling on edge $e(i, j)$ at time t . Their values vary as the traffic condition changes, but is unavailable in current vehicle navigation systems, since it depends on the real time measurement of vehicles traversing through edge $e(i, j)$.

Historical speed (HS), $s_H(t, e(i, j))$, time-dependent. It is the historical speed recorded and computed on edge $e(i, j)$ before time t . Taking aging factor into account,

$$s_H(t, e(i, j)) = \alpha \times s_R(t - \delta, e(i, j)) + (1 - \alpha) \times s_H(t - \delta, e(i, j))$$

Provisioning of the historical speed is based on availability and collection of the real-time mean speeds.

The path $p(S, D)$ is not unique, because for any departure time t , a vehicle started from n_S may have many different choices to reach its destination n_D . If strategy X is used for routing, the path is named as $p_X(S, D)$.

In order to measure the vehicle routing path, a cost function $c(t, e(i, j))$, associated with edge $e(i, j)$ needs to be defined. There exist various cost functions: i) traveling time, ii) traveling distance, iii) gasoline consumption, or others. If we use traveling distance to measure the cost, $c_D(t, e(i, j))$ equals to $d(e(i, j))$ and is invariant with time. On the other hand, if we use traveling time to measure the cost, $c_T(t, e(i, j))$ sets to be $d(e(i, j))/s_R(t, e(i, j))$ and varies with time. Note that the real-time speed is used here instead of the road speed since the latter cannot reflect the achievable speed under real-time traffic.

In this paper, the traveling time is assumed to be a major concern. The cost function for each edge, $c_T(t, e(i, j))$, is set to be the time spent when the vehicle actually traversed through the edge. Thus, if path $p_X(S, D)$ consists of m continuous edges, the total cost for a vehicle (departure at time t) to travel from n_S to n_D is defined as

$$\Psi_{X,t,S,D} = \sum_{e \in p_X(S,D)} c_T(t_k, e(n_k, n_{k+1})) \text{ for } 1 \leq k < m \text{ and } t_0 = t \\ \text{where, } c_T(t_k, e(n_k, n_{k+1})) = d(e(n_k, n_{k+1}))/s_R(t_k, e(n_k, n_{k+1})), \text{ and } t_{k+1} = t_k + c_T(t_k, e(n_k, n_{k+1})).$$

To minimize the total vehicle traveling time, an optimal routing path, $p_{OPT}(S, D)$ is defined such that

$$\Psi_{OPT,t,S,D} \leq (\Psi_{X,t,S,D}) \text{ for every routing strategy } X.$$

A. Routing algorithm description

In this section, we describe four vehicle routing algorithms with different traffic information. The first algorithm SPA, uses geographical information. The second algorithm OPT,

assumes that traffic information in the future is available, which is the optimal case for comparison. The third algorithm HBA, uses historical traffic information. The fourth algorithm ARA, adapts to the real time traffic conditions while routing.

Shortest path algorithm (SPA): The vehicle with SPA tries to find the lowest-weight path from source to destination. Here, the weight means ED, which is time-independent. Dijkstra algorithm is a typical approach to solve this single-source shortest path problem. SPA has been widely utilized by many commercial vehicle navigation systems.

Optimal algorithm (OPT): We construct the optimal routing path (with minimum travel time) for comparison purpose, although it is not practical. Assume that we can obtain and record the RTS for every edge. An algorithm in [10] can be applied to compute the optimal path, in which the basic idea is to adopt look-back approach (posterior checking) after we know all the RTSs.

Historical based algorithm (HBA): Instead of relying on the real-time speed, we use HS to achieve approximation routing. In reality, the historical speeds are generated in a coarse time interval, during which their real-time mean speeds do not change substantially. Typical time periods rely on the traffic patterns of road section $e(i, j)$, including weekdays or weekends, peak time or non-peak time, etc. Thus, we can record HS $s_H(t, e(i, j))$ for T_{PK} minutes during the peak time and T_{NP} minutes during the non-peak time. The weight of $e(i, j)$ is a function of HS $s_H(t, e(i, j))$ and ED:

$$s_H(t, e(i, j)) = \alpha \times s_R(t - \delta, e(i, j)) + (1 - \alpha) \times s_H(t - \delta, e(i, j)) \\ w_{HBA}(t, i, j) = d(e(i, j)) / s_H(t, e(i, j))$$

where α is the aging coefficient. We set T_{PK} to be 5 minutes and T_{NP} 10 minutes. Also, we take δ as a week because traffic is normally repeated weekly. The weight of edge is known in advance as long as the HS of the corresponding time interval is provided. Moreover, the algorithm is the same as the OPT.

Adaptive real-time algorithm (ARA): One major drawback of HBA is lack of real-time traffic information. We observed that the real time speed $s_R(t, e(i, j))$ sometimes can be substantial different from $s_H(t, e(i, j))$ as discussed in later section.

On the other hand, since we cannot obtain the real-time speed for the future, the optimal path $p_{OPT}(S, D)$ is impossible to be determined in real-time [9]. However, if the real-time traffic information can be known ahead, a vehicle can adapt the next step decision of routing at every intersection point. Here, the weight of $e(i, j)$ is a function of the real-time speed $s_R(t, e(i, j))$ and the edge distance:

$$w_{ARA}(t, i, j) = d(e(i, j)) / s_R(t, e(i, j))$$

Note that use this weight function to determine path $p_{ARA}(S, D)$ is different from that of $p_{OPT}(S, D)$. With arrival time t_k at intermediate node n_k to find the next hop, $w_{ARA}(t_k, i, j)$ will be used for every road section $e(i, j)$. Consequently, after traversing through the next hop with arrival time t_{k+1} at intermediate node n_{k+1} , $w_{ARA}(t_{k+1}, i, j)$ will be used again in determining the next hop. The process will continue until destination n_D is reached.



Figure 1. A source-destination path

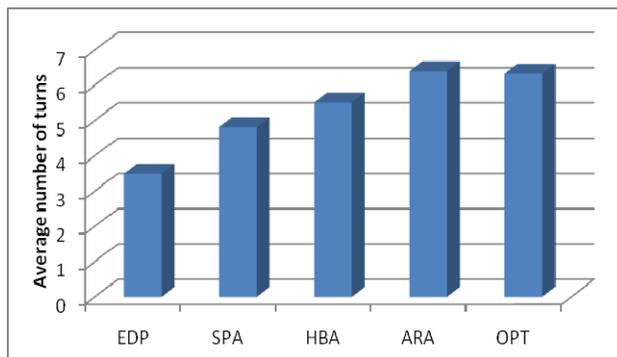


Figure 2. Average number of turns per path

IV. INFORMATION PROCESSING

In order to compare the quality of the vehicle routing algorithms proposed in section V, both geographical information and traffic information are required. In this work, we collected traffic information of vehicles from a vehicular CPS, called SUVNet. The GPS information collected from these vehicles included their timestamped geographical positions, speeds, and so on, which provide valuable information for evaluation of various routing strategies and for real implementation of an *intelligent vehicle routing system*.

GPS data collection and map matching: The GPS data have been collected from over 4,000 taxis in downtown area, the so-called “inner-loop area”. Data provided by those taxis is incomplete, yet imprecise. For example, a taxi reports about every 40-100 seconds and keeps silent during the interval. In order to reconstruct the trace of taxi from the sparse and imprecise data, we need three procedures to pre-process the collected data. First, we map the GPS data onto the digital road map. Second, we determine a route between two consecutive data samples. Third, more points need to be interpolated during the interval.

Real-time speed estimation: The GPS data are collected from individual vehicles and the road-wide traffic status is estimated using a mean speed. The road network of Shanghai is of large-scale and high complexity, approximately 1500 edges (road sections) and 1000 nodes (road intersections). Most of the edges are usually short ranging from 50m to 300m. Thus, two GPS data sampled consecutively from the same vehicle, in most cases, could reside in two different edges. Thus, it is required to aggregate data from multiple edges and multiple vehicles for an accurate estimation of traffic status.

In order to estimate a road traffic speed at time t , we utilize data collected from a group of vehicles. Specifically, the RTS can be estimated by vehicle-based algorithm (VBA) with GPS data, as described in our previous works [9]. After mapping and speed estimation, we obtained the speed information for every road, which provides the base for our performance study.

Empirical driving path (EDP): It is interesting to identify what kinds of routing paths were taken by taxi drivers and how different these paths are from those selected by other routing algorithms. We named these taxi paths as EDP because they are routed by taxi drivers based on their experience without help from navigation systems.

We use the following method to select 100 testing taxi paths. First, we select an area of 20km² in the downtown of Shanghai. On March 1, 2007, we have the GPS traces of about 4000 taxis in this area, from which 500 taxis with most GPS data were selected. For each taxi, there is a status bit piggy-backed with GPS data (1: busy, 0: idle or off-duty). When the status bit changes from 0 to 1, the source of a new path is marked and the departure time is recorded. The path is continuously traced until the destination is reached and the status bit switches back to 0. Here, we mainly focus on the on-duty paths because taxis will have definitely destinations instead of cruising on roads for picking up passengers. At the same time, not every path can be used to study the driver behaviors. We have identified about 900 complete paths. When these paths are mapped to the Shanghai city map, the routes of many paths still cannot be fully determined since the GPS data is imprecise yet incomplete. Finally, we selected 50 paths during peak time (7am to 11am and 3pm to 7pm) and other 50 paths during non-peak time. For each taxi, we only selected one path to guarantee the variety of our cases.

V. QUALITY EVALUATION

In this section, we evaluate the quality of four routing algorithms and EDP with the 100 source-destination pairs selected above. For every path with its departure time, four algorithms, SPA, OPT, HBA and ARA, are applied to find corresponding routing paths and compared them with the EDP path.

A. Analysis of typical paths

Here, we show some typical cases of routing algorithms. Figure 1 shows a source-destination pair with five paths. We found that EDP only had two turns; however, it traversed through a very congested road (RenMing road), which may explain why EDP took longer time. SPA selects a shorter path that traverses through another congested road (JiuJiang road and West NanJing road). HBA and ARA have chosen less congested roads since historical or real-time information guided them to avoid potential congestion. It is interesting to see that OPT took a different path, moving away from the jammed area and taking a longer distance but shorter time.

In fact, we found that EDP normally takes few turns, most likely because taxi drivers have a tendency not to change roads frequently. As illustrated in Figure 2, we compare different vehicle routing strategies in terms of their average number of turns per path.

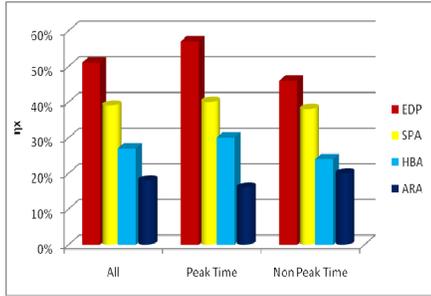


Figure 3. Comparison of extra traveling time

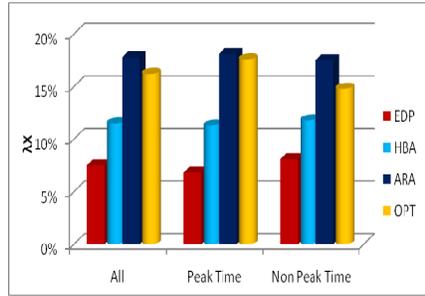


Figure 5. Comparison of extra traveling distance

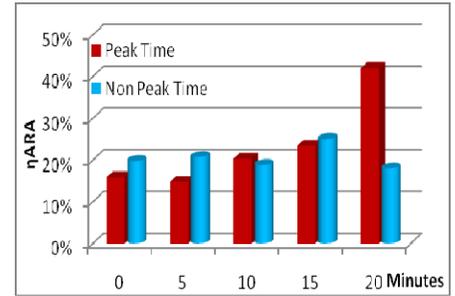


Figure 6. ARA routing quality

B. Routing quality

The OPT algorithm selects the optimal path with minimum traveling time, it is used for comparison. For evaluation purpose, routing quality η_X is defined to measure how close a routing algorithm towards the optimum, $\Psi_{OPT,t,S,D}$

$$\eta_X = (\Psi_{X,t,S,D} / \Psi_{OPT,t,S,D} - 1) \times 100\%$$

The smaller η_X is, the better the routing algorithm is.

Figure 3 shows the routing quality of EDP, SPA, HBA and ARA, compared to OPT, where ‘‘All’’ means all of 100 paths, ‘‘Peak Time’’ and ‘‘Non Peak Time’’ shows paths during the peak and non-peak time, separately.

To elaborate routing quality further, we present a pair-wise and a global comparison among these algorithms by recording the number of times each algorithm performed better, worse or equal in 100 test cases. This comparison is given in a graphical form shown in Figure 4. Here, each box compares two algorithms—the left one and the top one. A box contains three numbers preceded by ‘<’, ‘>’ and ‘=’ signs which indicate the number of times the left algorithm performed better, worse, or equal, than the top algorithm. For example, the ARA algorithm performed worse than HBA in 30 cases, better in 67 cases and the same in 3 cases. For global comparison, an additional box (‘‘ALL’’) for each algorithm compares that algorithm with all other algorithms combined. Based on these results, we rank these algorithms (or EDP trace) in the following order: ARA, HBA, SPA, and EDP. This ranking essentially indicates the routing quality in terms of travelling time.

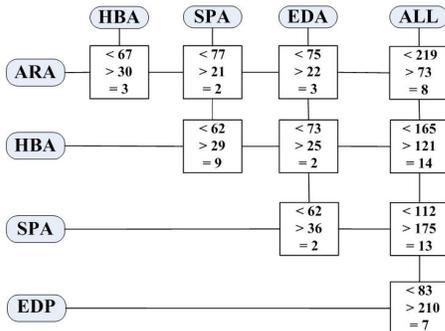


Figure 4. Comparison across 100 paths

Both routing quality comparison (in Figure 3) and routing algorithm global comparison (in Figure 4) consistently indicates that ARA performs the best and EDP the worst. More interesting observations are listed as follows.

- Taxi drivers: Different from our original expectation that taxi drivers could utilize their experience to find a reasonably good path, taxi drivers are in fact unable to find a path that is comparable to the paths generated by other methods. They

tend to select major roads and to make fewer turns. They attempt to avoid traffic congestion by their experience, but the detour is often congested as well. Thus, their choices end up being worse than SPA.

- Historical information: The historical traffic information supposed to be able to help drivers to make a right decision. It did help but not as much as expected. In our experiment, the difference between the RTS and the HS calculated by our formula is 61% in average. Although further improvement in manipulating the historical information may reduce this difference, the traffic speed usually has a large variation at real time and cannot be accurately predicted. Thus, it is not easy to solely rely on historical information to make a good choice.

- Real time information: Real time information is proved useful by our evaluation. Since the speed of roads changes smoothly at non-peak time, the real time information is believed to be particularly important for vehicle routing during the peak time. This fact has been further confirmed by our work. Especially, at peak time, ARA performs much better than HBA and outperforms EDP by almost 40%.

In terms of the path length (traveling distance), Figure 5 shows the routing comparison of EDP, HBA, ARA and OPT. The SPA algorithm is used as the comparison base as it provides the shortest traveling distance. In Figure 5, another routing quality λ_X is defined to measure how close that a routing algorithm can approach to the optimum, $TD_{SPA,t,S,D}$

$$\lambda_X = (TD_{X,t,S,D} / TD_{SPA,t,S,D} - 1) \times 100\%$$

where TD is the total traveling distance,

$$TD_{X,t,S,D} = \sum_{e(i,j) \in pX(S,D)} d(e(n_i, n_j))$$

The traveling distance is not sensitive to the traffic status, that is, it is about the same for peak time and non-peak time. By average, the distance of EDP path is no more than 10% extra compared to that of SPA paths as shown in Figure 5. Even the difference in distance is not large and taxi drivers may think their selected roads can be fast, EDP paths eventually take longer time as shown in Figure 3. Thus, though many taxi drivers claim that they do not need a GPS navigator, a navigator using the SPA algorithm indeed provides them a better driving path. Usually, HBA choose a longer distance path compared to EDP paths, but takes shorter traveling time. The traveling time of ARA is even shorter though its average distance is longer.

The results in this experimental study exhibit that real time information can be quite useful, especially during peak time. The history information is also useful when traffic is not heavily congested. We suggest that an appropriate level of traffic information should be provided for vehicle routing.

C. Influence of information delay in ARA

In ARA routing algorithm, it is assumed that every vehicle can get the real-time information without delay. However, it takes time to collect, to process and to disseminate. For a large-scale transportation system, such as Shanghai, real time information can be collected from millions of vehicles, spread over thousands of road segments. Therefore, information delay is inevitable. Figure 6 shows ARA routing quality when the time delay is 0, 5, 10, 15 and 20 minutes. In this figure, 5-minute delay with either heavy or light traffic is acceptable. During peak time, 10 to 15-minute delay may result in about 5% performance degradation; and 20-minute delay could make the real time information not useful since it performed worse than an algorithm that does not utilize real time information. Note that the performance of ARA routing at non-peak time does not vary substantially as the real time information delay increases. It is due to fact that traffic conditions changes slowly during non-peak time.

D. More Discussion

In this section, we discuss the quality evaluation of routing algorithms, our observations, as well as suggestions for the future work.

GPS navigator: Taxi drivers, and probably as well as other drivers, intend to make fewer turns, which may result in a sub-optimal decision. In addition, since many drivers choose the path with fewer turns, traffic may gather over those major roads to make other roads stay uncongested. Therefore, a GPS navigator using the SPA algorithm to find a route can be helpful to a driver. A navigator can reduce traveling time by about 10%.

Historical information: Historical information is useful for vehicle path routing. If we can provide vehicles with the HS for the past a few weeks, a better path can be computed. The benefit from this method could be more than 10% compared to a navigator with only road information.

Real-time information: Currently, many cities plan to establish an infrastructure to provide real-time road status, but few researches tell us how much benefit can be obtained from real-time information. With the quantitative evaluation presented in this article, we at least gain an insight into this kind of systems. Real-time traffic information is indeed useful to guide vehicles to find a less congested path. It brings about additional 10% improvement compared to the historical information. More importantly, the ARA algorithm is not only useful for individuals to save traveling time, but also beneficial to improve global transportation efficiency in terms of dispersing traffic. We also found that the ARA algorithm produces good routes which are only 15 to 20% worse than the optimal routes. Since the optimal route can only be computed after the vehicle reaches the destination, the ARA algorithm could be the best feasible choice for vehicle routing so far.

VI. CONCLUSION

This paper evaluate the quality of five vehicle routing algorithms with different level of traffic information on a real vehicular Cyber Physical System, called SUVNet, in Shanghai, which include more than 4000 taxis. We found that the performance of taxi drivers is not as good as expected. The traveling time of the path taken by the taxi driver is about 50% more than the optimal path and 30% more than that generated

by the ARA algorithm. Commercial GPS navigators and historical traffic information are useful to help vehicle drivers. Also, the delay of real-time traffic information substantially impact on performance of the ARA algorithm.

We suggest some research topics for future work. First, we propose to integrate ARA with HBA to design a new algorithm that performs closer to the OPT algorithm. Second, information locality is crucial for real deployment of real-time traffic system and need further study. Finally, when real-time traffic information is available to public, it may in turn affect the traffic itself, which is essentially beneficial for effective traffic control and management.

ACKNOWLEDGEMENTS

This research was partially supported by Natural Science Foundation of China grant No.60573138 and 60773091, the National Grand Fundamental Research 973 Program of China under Grant No.2006CB303000, the National 863 Program grant No.2006AA01Z247, National Science Foundation of USA grant CNR-0626380, and grant No. 2009ZX03006-001-01 from MIIT of China.

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