The Charging-Scheduling Problem for Electric Vehicle Networks

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***Abstract*—Electric vehicle (EV) is a promising transportation with plenty of advantages, *e.g.*, low carbon emission, high energy efficiency. However, it requires frequent and long time charging. In public charging stations, EVs spend long time on queuing especially during peak hours. Hence, it requires an efficient method to reduce the total charging time for EVs. We study the Electric Vehicle Charging-Scheduling (EVCS) problem in this paper. First we prove that EVCS is NP-Complete, which can be reduced from one Parallel Machine Scheduling (PMS) problem. Then two heuristic algorithms are proposed: the Earliest Start Time (EST) algorithm, and the Earliest Finish Time (EFT) algorithm. EST tries to advance the start charging time to get customers in service as early as possible, while EFT focuses on the possible finish charging time to get customers served as soon as possible. Finally simulations show that, the proposed algorithms outperform the classic greedy nearest scheduling algorithm: assign each EV to its nearest charging station, then choose the outlet where the fewest EVs are queuing. Typically, under our simulation settings, the average finish time and maximum finish time can be reduced by about one hour, and six hours respectively.**

I. Introduction

Electric vehicles (EVs), *e.g.*, electric motorcycles, electric boats, and electric cars, spring up in plenty of countries recent years. They possess several advanced aspects such as low carbon emission, and high energy efficiency. President Obama has announced a budget of $2.4 billion to develop the next generation of EVs [1]. China promotes environment-friendly EV transportation by subsidizing buyers [2].

The capacity of EV battery is limited, which requires frequent charging. EV battery swapping is well known and only takes a few minutes, but unfortunately EV manufacturers have different standards for battery access, attachment, and type. Although a great work has been accomplished to reduce the charging time, it remains still quite long, *e.g.*, six to ten hours.

Charging stations are one of the most popular charging places, apart from homes and workplaces. EV manufacturers, charging station providers, and governments have signed plenty of agreements to promote electric vehicle ad-hoc networks. If public charging stations are not well deployed, the total time spent on charging would be dramatic [3]. The limited number of charging stations together with the long recharging time require a better EV scheduling in order to reduce the total amount of time spent on charging.

Recently lots of researches have been conducted regarding the charging-scheduling problem for EVs. Qin et al. propose a distributed scheme to minimize the waiting time of EVs [4]. Li et al. introduce a distributed optimal EV charging algorithm in cyber-physical systems [5]. It focuses on EV integration in power grids. To minimize the distribution system load variance, authors propose a decentralized smart plug-in electric vehicle (PEV) charging algorithm [6]. Gan et al. propose an optimal decentralized protocol to schedule EV charging, which exploits the elasticity of EV loads to fill the valleys in electric load profiles [7]. In [8], charging stations decide the charging order by a linear rank function, which is based on the estimated arrival time, the waiting time bound, and the amount of demanded electricity. Steen et al. study an approach to decide when and where plug-in EVs can be charged, and it is based on demographical statistical data [3]. He et al. propose a globally optimal scheduling scheme and a locally optimal scheduling scheme for EV charging and discharging [9]. Kempton et al. introduce a vehicle-to-grid (V2G) system [10], where EVs deliver electricity into the grid or throttle their charging rate.

Different from the above studies, our work focuses on reducing the total time spent on charging when EVs demand charging on a road network, which includes the travel time on roads, the queueing time, and the actual charging time. In this paper, we build a system model for the EV charging scheduling, and formulate the Electric Vehicle Charging-Scheduling (EVCS) problem. Then we prove EVCS is NP-Complete, which can be reduced from one kind of PMS problems, which is known to be strongly NP-Hard. Then we propose two heuristic algorithms for EVCS: EST and EFT. EST tries to advance the start charging time to get customers in service as early as possible, and it is able to reduce the idle time between the EVs. EST ignores the actual charging time, while EFT takes it into account, which focuses on the possible finish charging time to get customers served as soon as possible. Through the simulations, the proposed algorithms outperform one classic greedy nearest scheduling algorithm.

The rest of this paper is organized as follows: Section II presents the system models and defines the EVCS problem. In Section III, we prove it to be an NP-Complete problem. In Section IV, two heuristic algorithms are proposed. In Section V, we compare their performance with one classic

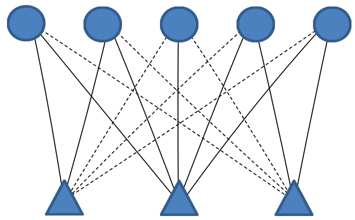

Fig. 1. A graph constituted by EVs, charging stations, and road networks.

greedy algorithm, and the simulations show that the proposed algorithms exhibit better performance. Section VI concludes this work.

II. System Models and Problem Formulation

This section presents the system models for the EVCS problem and formulates it to be an optimization problem.

A. Electric Vehicle Network Model

Assuming that on a road network, there are y charging stations $Y = \{1, 2, \ldots y\}$ and n EVs $I = \{1, 2, \ldots n\}$ which require battery charging, where $y \geq 2, n \geq 2$. Each charging station has q charging outlets. Therefore, there are $m = y * q$ charging outlets $J = \{1, 2, \ldots m\}$. So the EVs, charging stations, and the roads form a graph $G =< \nu, \varepsilon >$, where ν represents a set of EVs and charging stations, and ε represents the set of roads that connect EVs and charging stations. Fig. 1 shows a graph constituted of EVs (denoted by circles), charging stations (denoted by triangles), and road networks (denoted by edges). If the EV can reach the charging station, the edge is denoted using solid line, otherwise, using dotted line. The weight of each edge is the shortest distance between the EV and the charging station.

Suppose the charging stations are identical, thus the charging rate e_i^+ (A) of EV i are identical for all charging stations. The shortest distance between EV i and charging outlet j is denoted by $d_{i,j}$ (Km), which can be calculated from the graph G through the shortest path algorithm (*e.g.*, Floyd Warshall Algorithm). For any EV, the distance to the charging outlets in the same charging station is identical. Therefore, we only calculate once. All the EVs are equipped with wireless communication devices, such that all the information (*e.g.*, velocity, the rate of electricity consumption, the remained electricity, location) is known to the central server so as to improve the scheduling decisions. Each EV only chooses one charging outlet for charging until it is fully charged, which cannot be interrupted during the actual charging time. The queuing sequences of the EVs are determined by their arrival time in any charging outlet, *i.e.*, there is no preemption. Moreover, we assume that with the remained electricity each EV is able to reach at least one charging station, *i.e.*, q charging outlets.

To facilitate the modelling of this system, the following notations are introduced:

- η_i (Ah): the maximum battery capacity of EV i. In the market, it is determined by materials, the number of battery packs, etc.
- e_i^- (A): the rate of electricity consumption per unit time of EV i. Generally the rate of electricity consumption is denoted using the maximum electricity capacity. *e.g.*, the battery capacity of an electric car is 54Ah, and it consumes electricity at a rate of 5.4A, then we get that it consumes electricity at a rate of $0.1\eta_i$.
- $a_{i,j}$ (h): the time EV i arrives in charging outlet j. From our assumption, the arrival time of any EV in the charging outlets of one charging station is identical.
- e_i (Ah), $e'_{i,j}$ (Ah), b_i (Ah): the initially remained electricity of EV i is e_i, and the remained electricity when it reaches the charging outlet j is $e'_{i,j}$. Both of them have a lower bound b_i, *i.e.*, at any time each EV ensures it should have at least b_i electricity. If the EV runs out of energy, the battery life would be greatly shortened. Of course, $e_i \geq b_i, e'_{i,j} \geq b_i$. We get $e'_{i,j} = e_i - a_{i,j} * e_i^-$.
- ζ_i (Km): the maximum distance the EV i is able to travel before the electricity reaches its lower bound, which is related to the initially remained electricity.
- $c_{i,j}$ (h): the actual charging time of EV i if it chooses charging outlet j.
- v_i (Km/h): the velocity of EV i. Generally, the rate of electricity consumption is greater, the velocity is greater. And it is related to the weight of the EV and driver.
- τ (h): In certain charging outlets there have been some EVs waiting for charging in advance. For charging outlet j, the finish charging time of the last one among the originally existed EVs is denoted by τ_j. Let $\tau = \{\tau_1, \tau_2, \ldots, \tau_m\}$. Here, we assume it follows a Possion process, $\tau \sim P(\lambda)$.

If we schedule EVs to charging stations, then we would also decide which charging outlets they should select. In this paper, we directly schedule EVs to charging outlets. Considering the relationships between the EVs and the charging outlets, and the relationships between the EVs aiming at the same charging outlet, the following 0-1 variables are defined ($\forall i, k \in I, j \in J$):

$$\delta_i^j = \begin{cases} 1, & \text{if EV } i \text{ can reach charging outlet } j, \\ & \textit{i.e.}, d_{i,j} \leq \zeta_i \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$x_i^j = \begin{cases} 1, & \text{if EV } i \text{ chooses charging outlet } j \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$$x_{0,i}^j = \begin{cases} 1, & \text{if EV } i \text{ is the first in charging outlet } j \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$x_{k,i}^j = \begin{cases} 1, & \text{if EV } i \text{ is immediately after} \\ & \text{EV } k \text{ in charging outlet } j \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

$$P_i^j = \{k \in I \backslash \{i\} | \text{ EV } k \text{ can precede EV } i \text{ in charging outlet } j \ \} \quad (5)$$

The maximum distance ζ_i, the arrival time $a_{i,j}$, and the actual charging time $c_{i,j}$ of EV i (if it can reach and chooses charging outlet j) can be respectively obtained using Eqn(6)(7)(8). From our assumption, for any EV i and the charging outlets in one charging station, the arrival time and the actual charging time are identical. Therefore, we only calculate once.

$$\zeta_i = v_i \frac{e_i - b_i}{e_i^-} \tag{6}$$

$$a_{i,j} = \frac{d_{i,j}}{v_i} \tag{7}$$

$$c_{i,j} = \frac{\eta_i - e'_{i,j}}{e_i^+} = \frac{\eta_i - e_{i,j} + a_{i,j} e_i^-}{e_i^+} \tag{8}$$

B. Problem Formulation

The EVCS problem is to schedule the EVs to appropriate charging outlets for charging such that the total time spent on charging is minimized, which includes the travel time to the charging outlets, the queuing time, and the actual charging time. Obviously, if we ignore the queuing time of EVs in charging outlets, the problem is maximum bipartite matching. While in the real world, there are several constraints to consider. *e.g.*the electricity left in the battery prevents an EV from reaching some charging outlets. If there are plenty of EVs queuing in advance for charging in certain charging outlets, then the total time is increased due to the time spent waiting in the line. The charging rate of a charging outlet is limited, which leads to long time for actually recharge a battery. Given all the above constraints, we should develop an appropriate scheduling decision for each EV.

To clearly describe EVCS, we consider the following s-cenario: for any single charging outlet j, let V_j ($V_j \subseteq I$ and $N_j = |V_j|$) denote the EVs that choose the charging outlet j for charging. We get $n = \sum_{j=1}^{m} N_j$. Let $V_{l,j}$ denote the lth EV of V_j ($l \in \{1, 2, \ldots, N_j\}$). The arrival time $\{\hat{a}_{1,j} \leq \hat{a}_{2,j} \ldots \leq \hat{a}_{N_j,j}\}$ and the actual charging time $\{\hat{c}_{1,j}, \hat{c}_{2,j}, \ldots \hat{c}_{N_j,j}\}$ are deterministic. Let $\hat{\phi}_{l,j}$ denote the finish charging time of $V_{l,j}$, and ϕ_j denote the sum of the finish time of each EV in V_j, *i.e.*, $\phi_j = \sum_{l \in \{1,2,\ldots,N_j\}} \hat{\phi}_{l,j}$. Then we obtain Eqn(9).

$$\hat{\phi}_{l,j} = \begin{cases} max(\tau_j, \hat{a}_{1,j}) + \hat{c}_{1,j}, & \text{if } l = 1 \\ max(\hat{\phi}_{l-1,j}, \hat{a}_{l,j}) + \hat{c}_{l,j} & \text{otherwise} \end{cases} \tag{9}$$

Let t_i denote the finish charging time of EV i, and it can be obtained by Eqn(10).

$$\begin{aligned} t_i = &\sum_{j \in J} \{(max(\tau_j, a_{i,j}) + c_{i,j}) x_{0,i}^j + \\ &\sum_{k \in P_i^j} (max(t_k, a_{i,j}) + c_{i,j}) x_{k,i}^j\} \end{aligned} \tag{10}$$

Our problem can be formulated as (11)(12)(13)(14), which reduce the scheduling problem to a 0-1 integer programming (IP) problem, or as an assignment problem. (11) is the objective function, where t_i is calculated by Eqn(10), and (12)(13)(14) are subjection. (12) ensures that each EV is scheduled once and it is capable to reach the chosen charging outlet. (13) ensures that any EV at most has one precursor in any charging outlet. (14) means that if an EV is the first one in a charging outlet, it is impossible to appear in other charging outlets.

$$\text{Objective: minimize} \sum_{i \in I} t_i \tag{11}$$

Subject to:

$$\sum_{j \in J} x_i^j \delta_i^j = 1 \tag{12}$$

$$\sum_{j \in J} \sum_{k \in P_i^j \bigcup \{0\}} x_{k,i}^j = 1 \tag{13}$$

$$\sum_{i \in I} x_{0,i}^j \leq 1 \tag{14}$$

$$x_i^j, x_{k,i}^j, x_{0,i}^j \in \{0, 1\}.$$
$$\forall i, k \in I, j \in J.$$

III. Complexity Analysis

Section II describes the system models and formulates the EVCS problem. This section analyzes the complexity of EVCS, and proves it to be NP-Complete.

A. NP-Completeness

Theorem III.1. *The Electric Vehicle Charging-Scheduling (EVCS) problem is NP-Complete, as it can be reduced from one kind of Parallel Machine Scheduling (PMS) problems:* $R|r_j|\sum \omega_j C_j$*, which is strongly NP-Hard.*

Proof: First, we show that EVCS belongs to NP. Given any instance of EVCS, we use as a certificate the schedule S of the EVs. The verification algorithm checks that any EV $i \in I$ is assigned to one charging outlet $j \in J$, then sums up the finish charging time of each EV and checks whether the sum is at most T. Obviously, the process can be done in polynomial time ($O(|I||J|)$). So EVCS is NP.

Second, we prove that EVCS is NP-Hard by showing that it can be reduced from one kind of Parallel Machine Scheduling (PMS) problems [11], $R|r_j|\sum \omega_j C_j$ [12] [13], which is known to be strongly NP-Hard. There are plenty of PMS problems, a large number of which are NP-Hard. For $R|r_j|\sum \omega_j C_j$, R indicates the unrelated machines, *i.e.*, each machine processes different jobs at different speed. r_j indicates the release time of job j, *i.e.*, the earliest time which can start its processing. $\sum w_j C_j$ is the objective function, which denotes the sum of completion time of each job. Where, w_j denotes the weight of job j, and C_j indicates the completion time of job j. $R|r_j|\sum \omega_j C_j$ shows such a scenario: There are n jobs, m machines, and each job has to be processed on just one of the m unrelated machines. Each job has a processing time, while the processing speed of the machines are not identical. The actual processing time $p_{i,j} = p_j / s_i$, where p_j denotes the processing time of job j, and s_i denotes the processing speed of machine i. Each job is assigned once to one machine.

To describe briefly, let PMS substitute PMS-$R|r_j|\sum \omega_j C_j$. Each job in PMS is mapped to an EV in EVCS, and each machine in PMS is mapped to a charging outlet in EVCS. The processing time of each job in each machine in PMS is mapped to the charging time of each EV in each charging outlet of EVCS. The release time of each job in PMS is mapped to the arrival time of each EV to charging outlets in EVCS.

In PMS, there are no precedence constraints between the jobs. While in EVCS, the precedence of the EVs are different for different charging outlets, which are determined by the arrival time of the EVs, and the fact that some EVs cannot reach some charging outlets. The release time of each job in PMS is deterministic, and is identical to each machine. In EVCS the earliest start charging time of any EV is determined by both the arrival time and the queuing time. The arrival time is calculated by Eqn(7), and the queuing time is determined by the scheduling decisions, which differs in different charging outlets.

Therefore, for any instance of PMS, a corresponding instance in EVCS is mapped to it, and the reduction is polynomial. And the solution to the instance in EVCS is also the solution to PMS. As a result, EVCS is NP-Hard. A problem is NP-Complete if it is NP and also is NP-Hard. Thus, EVCS is NP-Complete. ■

IV. Algorithm

This section reformulates the EVCS problem and proposes two heuristic algorithms.

A. Heuristics

The formulation of the EVCS problem using assignment variables (*i.e.*, x_i^j, $x_{k,i}^j$, $x_{0,i}^j$) suggests a natural decomposition of the problem into a set of one single charging outlet scheduling problems. Therefore, it is convenient to rewrite the objective function: $\sum_{j\in J}\phi_j$, and ϕ_j can be obtained using $t_i (i \in I)$ through Eqn(15).

$$\phi_j = \sum_{i\in I} t_i x_i^j \tag{15}$$

A jump function $U(a,b)$ is defined by Eqn(16), through which the objective function can be described by Eqn(17).

$$U(a,b) = \begin{cases} a-b, & \text{if } a > b \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

$$\sum_{i\in I} t_i = \sum_{j\in J}\{N_j * \hat{\phi}_{1,j} + (N_j - 1) * (U(\hat{a}_{2,j}, \hat{\phi}_{1,j}) + \hat{c}_{2,j}) + \ldots + U(\hat{a}_{N_j,j}, \hat{\phi}_{N_j-1,j}) + \hat{c}_{N_j,j}\} \tag{17}$$

Given any single charging outlet j, for $l \in \{1,2,\ldots,N_j\}$ let $h_{l,j}$ denote the time period between the finish time of the lth EV and its predecessor in the same charging outlet j, which is shown by Eqn(18). Let h_j denote $\{h_{1,j}, h_{2,j}, \ldots, h_{N_j,j}\}$. We can reformulate the objective function through Eqn(19).

$$h_{l,j} = \begin{cases} \hat{\phi}_{1,j}, & \text{if } l = 1 \\ \hat{\phi}_{l,j} - \hat{\phi}_{l-1,j}, & \text{otherwise} \end{cases} \tag{18}$$

$$\sum_{i\in I} t_i = \sum_{j\in J}\phi_j = \sum_{j\in J}\{N_j * h_{1,j} + (N_j-1) * h_{2,j} + \ldots + 2 * h_{N_j-1,j} + h_{N_j,j}\} \tag{19}$$

From Eqn (19), we know that the N_j coefficients $\{N_j, N_j - 1, \ldots, 2, 1\}$ would to be assigned to N_j EVs. To minimize ϕ_j, assign the highest coefficient N_j to the smallest element of h_j, and the second highest coefficient $N_j - 1$ to the second smallest element of h_j, and so on. Note that $h_{l,j}$ is determined by the arrival time, the actual charging time of EVs, and the idle time between them. In addition, the sequence is determined by the arrival time of the EVs aiming to the same charging outlet, not arbitrarily.

Next, we propose two lemmas which show the relationships between the actual charging time and the finish charging time in any single charging outlet.

Lemma IV.1. *For any charging outlet j, $k,l,N \in \{1,2,\ldots,N_j\}$, and $k \le l \le N$, we get:*

$$\sum_{l=1}^{N}\hat{c}_{l,j}\hat{\phi}_{l,j} \ge \frac{1}{2}\{(\sum_{l=1}^{N}\hat{c}_{l,j})^2 + \sum_{l=1}^{N}(\hat{c}_{l,j})^2\} \tag{20}$$

Proof: From $\hat{\phi}_{l,j} \ge \sum_{k=1}^{l}\hat{c}_{k,j}$, we get:

$$\hat{c}_{l,j}\hat{\phi}_{l,j} \ge \hat{c}_{l,j}\sum_{k=1}^{l}\hat{c}_{k,j} \tag{21}$$

Summing for all $l \in \{1,2,\ldots,N\}$, $\sum_{l=1}^{N}\hat{c}_{l,j}\hat{\phi}_{l,j} \ge \sum_{l=1}^{N}\hat{c}_{l,j}\sum_{k=1}^{l}\hat{c}_{k,j}$. Clearly, we get the following:

$$\sum_{l=1}^{N}\hat{c}_{l,j}\sum_{k=1}^{l}\hat{c}_{k,j} \ge \frac{1}{2}\{(\sum_{l=1}^{N}\hat{c}_{l,j})^2 + \sum_{l=1}^{N}(\hat{c}_{l,j})^2\} \tag{22}$$

At last, we obtain (20). ■

Lemma IV.2. *For any charging outlet j, $k,l \in \{1,2,\ldots,N_j\}$ and $k \le l$, we get:*

$$\sum_{k=1}^{l}\hat{c}_{k,j} \le 2\hat{\phi}_{l,j} - \frac{\sum_{k=1}^{l}(\hat{c}_{k,j})^2}{\sum_{k=1}^{l}\hat{c}_{k,j}} \tag{23}$$

Proof: From Lemma IV.1, we get the following:

$$\hat{c}_{l,j}\hat{\phi}_{l,j} \ge \frac{1}{2}\{(\sum_{k=1}^{l}\hat{c}_{k,j})^2 + \sum_{k=1}^{l}(\hat{c}_{k,j})^2\} - \sum_{k=1}^{l-1}\hat{c}_{k,j}\hat{\phi}_{k,j} \tag{24}$$

For $\hat{\phi}_{k,j} \le \hat{\phi}_{l,j}$, we get (23). ■

The total time each EV spent on charging is split into three parts: the travel time (arrival time), the queuing time, and the actual charging time. By reducing the arrival time, we try to find the nearest charging outlet, so the actual charging time would be shortened. To reduce the queuing time, we should better choose a charging outlet where few vehicles are waiting for charging. Through Eqn(19), we know that N_j is not deterministic and that the idle time between the EVs in the same charging outlet can be reduced.

Let π_i^j denote the predecessor of EV i in the same charging outlet j, *i.e.*, $\pi_i^j = \{k | k \in I\backslash\{i\}, x_{k,i}^j = 1\}$. Denote $\mu_{i,j}$ as the earliest start charging time of EV i in charging outlet j, which can be obtained through Eqn(25).

$$\mu_{i,j} = \begin{cases} max(\tau_j, a_{i,j}), & \text{if } x_{0,i} = 1 \\ max(t_{\pi_i^j,j}, a_{i,j}), & \text{otherwise} \end{cases} \tag{25}$$

B. Algorithm

We propose two algorithms, the Earliest Start Time (EST) algorithm, and the Earliest Finish Time (EFT) algorithm. EST tries to advance the start charging time to get customers in service as early as possible, which is able to reduce the idle time between EVs. EST aligns the currently earliest start charging time in a nondecreasing order each EV to each charging outlet, among which EST chooses the earliest one. Such that one of the EVs is scheduled to one charging outlet which provides the earliest start charging time. Then EST inserts idle time as necessary if the arrival time of an EV is earlier than the finish time of its predecessor in the same charging outlet. Once an EV is scheduled, EST updates the earliest start charging time of the EVs which have not been scheduled. For the just scheduled EV is inserted in the queuing line, the predicted earliest start charging time of others should be changed. Then EST updates this scheduling order repeatedly until all the EVs are scheduled.

EST only focuses on earliest start charging time, thus it ignores the actual charging time. EFT focuses on the possible finish charging time to get customers served as soon as possible, which includes the actual charging time. EFT can be get through replacing the earliest start charging time of EST by the earliest finish charging time. EFT schedules the EVs in a nondecreasing order of the possible earliest finish charging time each EV to each charging outlet.

1) Algorithm of EST: The pseudo-code of EST is showed by Algorithm 1. Line 12 need n times, and line 13 need n^2m times. So the complexity is $O(n^2m)$.

Lines 1-5 determine the charging outlets that each EV is capable to reach. Lines 6-10 calculate the earliest start time of each EV if no EVs are scheduled using Eqn(25). Lines 12-15 find the earliest start charging time of the EVs which have not been scheduled. Line 16 determines the schedule decision of one EV. For an EV is scheduled to one charging outlet. The earliest start charging time of other EVs has to be delayed, which is functioned through lines 17-21. Line 22 means that the just scheduled EV should not be scheduled once more.

2) Algorithm of EFT: Let $f_{i,j}$ denote the possible finish charging time of EV i in charging outlet j. EFT can be modified from EST. Replace the start charging time $\mu_{i,j}$ in EST by the possible finish charging time $f_{i,j}$, and the equation used in EFT is Eqn(9). The complexity of EFT is also $O(n^2m)$.

Algorithm 1 Algorithm of EST

Input:

The arrival time of each EV to each outlet, $\{a_{i,j}\}$.
The charging time of each EV to each outlet, $\{c_{i,j}\}$.
The maximum distance each EV can travel, $\{\zeta_i\}$.
Where, $i \in I, j \in J$.

Output:

Scheduling decisions of all the EVs S.

1: **for** $i \in I, j \in J$ **do**
2:   **if** $d_{i,j} > \zeta_i$ **then**
3:     $\mu_{i,j} \leftarrow null$
4:   **end if**
5: **end for**
6: **for** $i \in I, j \in J$ **do**
7:   **if** $\mu_{i,j} \neq null$ **then**
8:     Calculate the earliest start time $\mu_{i,j}$ by Eqn(25).
9:   **end if**
10: **end for**
11: **repeat**
12:   **for** $i \in I, j \in J$ **do**
13:     Find the earliest start time $\mu_{min} \leftarrow \mu_{i',j'}$.
14:     If we find some EVs which have identical earliest start time, select the EV which has the earliest arrival time.
15:   **end for**
16:   Schedule EV i' to charging outlet j' .
17:   **for** $j \in J$ **do**
18:     **if** $\mu_{i,j} \neq null$ **then**
19:       Update the possible earliest start time μ' of the EVs which have not been scheduled by Eqn(25).
20:     **end if**
21:   **end for**
22:   **for** $j \in J$ **do** $\mu_{i',j} \leftarrow null$ **end for**
23: **until** All the EVs are scheduled

TABLE I
CONFIGURATIONS OF SIMULATOR

Item	Value	Item	Value
n	100, 150, 200, 250, 300	y	30
q	3	$d_{i,j}$	$4 \sim 30$
η_i	$20 \sim 80$	v_i	$2 \sim 3e_i^-$
e_i	$30\% \sim 45\%\eta_i$	e_i^+	$25\% \sim 30\%\eta_i$
e_i^-	$10\% \sim 15\%\eta_i$	b_i	$5\% \sim 10\%\eta_i$
λ	5	n_s	50

V. EVALUATION

In this section, we compare the proposed algorithms EST and EFT with one classic greedy scheduling algorithm.

A. Compared Algorithm and Metrics

From our assumption, each EV is able to reach at least one charging station. The proper algorithm to compare our work is the Nearest Algorithm (NA): Schedule each EV to its nearest charging station, then choose the outlet where the fewest EVs are queuing. The configurations of the simulator is listed in Table I. Let n_s denote the number of times the simulations run. Assume all the datas in their range follow a uniform distribution. *e.g.*, the capacity electricity of EVs is $20 \sim 80$ Ah, which follows a uniform distribution. The distance between each EV to each charging station is $4 \sim 30$ Km. We assume the velocity is in direct ratio to the rate of electricity consumption. Here, we set $v_i = 2 \sim 3e_i^-$.

B. Results

For the three algorithms EFT, EST and NA, we compare the average finish time, the maximum finish time, and standard deviation of finish time for the EVs. Table II shows the simulation results, where the number of EVs is 100. The

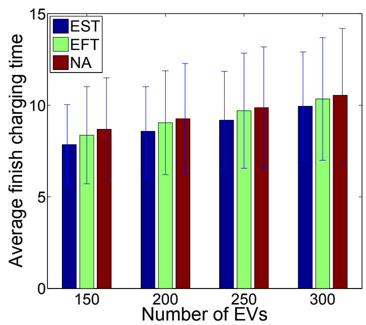

Fig. 2. Average finish time and standard deviation.

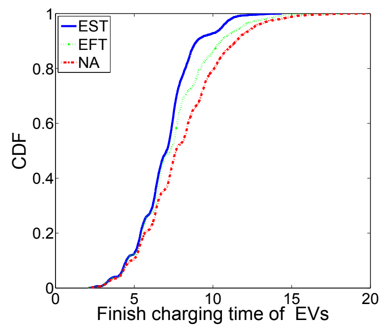

Fig. 3. The CDF of EST, EFT, and NA (100 EVs).

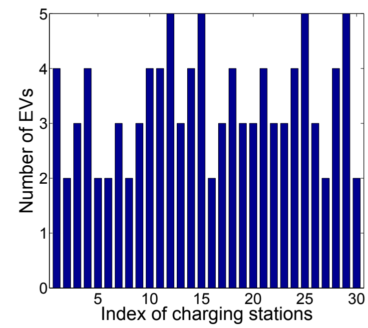

Fig. 4. Number of scheduled EVs in each charging station using EST (100 EVs).

TABLE II
RESULTS OF EST, EFT, AND NA (100 EVS)

Algorithm	Average time	Maximum time	Standard deviation
EST	6.95	13.46	1.83
EFT	7.43	18.10	2.32
NA	8.01	20.13	2.61

average time of EST and EFT are 1.06 hour (63.6 minutes) and 0.58 hour (34.8 minutes) shorter than NA, reduced by 13.2% and 7.2% respectively. The maximum finish time of EST is 6.67 hours shorter than NA.

Fig. 2 shows the average finish charging time and the standard deviation using EST, EFT and NA. The standard deviation of the finish time using EST and EFT are smaller than using NA.

Fig. 3 presents the CDF of the finish charging time using EST, EFT, and NA. We observe that the finish charging time using EST and EFT are smaller than using NA. More than 90% of the EVs using EST are capable to finish charging in ten hours, while if using NA, about 20% of the EVs have to spend more than ten hours. The maximum finish time of NA is about a little longer than EFT, but it is much longer than EST. The performance of EST is better than EFT. Fig. 4 shows the number of the scheduled EVs in each charging station using EST.

VI. CONCLUSION

In this paper, we formulate the EVCS problem, and build the system models. Then we prove that EVCS is NP-Complete. It can be reduced from one kind of PMS problems: $R|r_j|\sum \omega_j C_j$, which is strongly NP-Hard. We reformulate the objective function and propose two heuristic algorithms, EST and EFT. We compare the proposed algorithms with one classic greedy algorithm NA. Through Table II, Fig. 2 and Fig. 3 we conclude that, the performance of EFT and EST (especially EFT) are better than NA, and the average finish time and maximum finish time both can be reduced. The scheduling algorithms in this paper are offline. In the future, we plan to research on the online and dynamic charging-scheduling of EVs. The following optimization requires to be considered: the maximum lateness with respect to the deadline, the total tardiness, and the number of tardy EVs.

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