An Algorithm of Lane Change Using Two-Lane NaSch Model in Traffic Networks

Ming Zhu*, Jiajun Hu*, Linghe Kong†*, Ruimin Shen*, Wei Shu‡*, Min-You Wu*
*Shanghai Jiao Tong University, China
†Singapore University of Technology and Design, Singapore
‡University of New Mexico, USA
*{zhumingpassional, jiajun.hu, linghe.kong, rmshen, shu, mwu}@sjtu.edu.cn

***Abstract*—Few researches focus on lane change decisions achieving space or time optimality. In order to improve traffic efficiency, this paper studies the lane change problem. The goal of the lane change problem is to reduce the additional road space before all vehicles change to their target lanes. The widely adopted Nagel-Schreckenberg (NaSch) model is utilized, which reflects plenty of natural characteristics of actual traffic. An optimal lane change algorithm is proposed. Simulation shows that our approach outperforms the random lane change method. Typically, the performance is improved by 77.4% when the density of vehicles is 80% and the percentage of vehicles that requires lane change is 100%.**

I. Introduction

In the past few years, how to transport more vehicles to their destinations exactly and how to reduce traffic congestion are common problems in traffic networks [1]. There are two operations of the vehicles in roads: lane following and lane change. There are a lot of researches on lane following, while few on lane change. The drivers expect that the vehicles travel as fast as possible, while there are several constraints to consider, *e.g.*, safety, limited velocity, and limited acceleration. Recent years platoon maneuvers are paid attention, which cover platoon splitting, platoon merging, entry point, exit point, etc. Plenty of researchers propose the operation of lane change within or among platoons to improve traffic flux [2]. While few researchers focus on the road space optimization caused by the effects of lane change or traffic congestion.

Even each vehicle is able to obtain all the information of others, it is difficult to make a lane change decision without any coordination with others. If a decision is taken rashly without considering the mobility of other vehicles, there may occur traffic accidents. Under the coordination of the mobility of other vehicles, we are capable to obtain better lane change decisions. Such that the road capacity and traffic efficiency are improved, and traffic accidents are reduced.

This paper aims to reduce the additional road space before all vehicles change to their target lanes, and at the same time, improve the velocity of vehicles as far as possible. The problem can be divided into several subproblems considering of lane change in each row in a discrete road.

The remaining of the paper is organized as follows: Section II discusses related work. The models and assumptions are described in Section III. Section IV introduces an optimal operation of lane change in one row in a discrete road. And in Section V, a lane change algorithm for the whole vehicles is proposed, and we prove that it is optimal. In Section VI, the paper shows the computer simulations and results, and we analyze the performance of the algorithm. Section VII describes the conclusion of this work and some future work.

II. Related Work

In our real life, not only the lane change maneuver of one single vehicle, but also a Platoon Lane Change (PLC) is necessary. H. Hsu *et al*. propose two kinds of PLC maneuver: leader PLC maneuver and predecessor PLC maneuver [3]. The leader PLC maneuver is performed for an entire platoon to change lanes under a coordinated-platooning infrastructure. The vehicles perform lane change at the same time with the same trajectory. The predecessor PLC maneuver is designed in noncoordinated-platooning environments. The vehicles perform lane change from a leader vehicle to its followers one by one with a delay of communication time.

In order to improve traffic throughput of highway vehicles, and to ensure the vehicles exit successfully at their destinations, [4] presents an approach of lane assignment. Vehicles are organized into platoons for safety and great lane capacity. While the approach only provides the destinations for platoons, and it does not consider which lanes are the best options when the vehicles reach their destinations. In addition, it is difficult to operate the vehicles in one or several platoons with different target lanes.

The research on the Optimal Lane Reservation (OLR) [5] proposes a method to select the optimal lanes reserved in traffic networks. Almost all the lane change maneuvers should take into account the Minimum Safety Spacing (MSS) [6]. In [7], J. Hossein *et al*. point out that, given a particular lane change/merge scenario, the minimum longitudinal spacing can be calculated, such that collisions could be avoided, and some warnings for drivers or passengers can be provided. [8] finds that, the power spectrum of the traffic current presents the 1/f behavior [9] in certain parameter regions, which is the result of the interactions of clustering and hopping between vehicles.

III. Models and Assumptions

There are a large number of dynamic models to study traffic networks, *e.g.*, the Cellular Automaton (CA) model [10], the Nagel-Schreckenberg model [11], the LWR model [12], and

978-1-4799-2491-2/13/$31.00 ©2013 IEEE

DOI 10.1109/ICCVE.2013.99

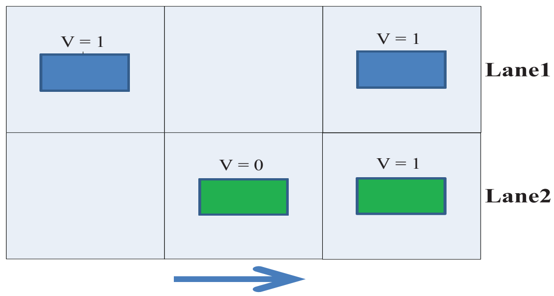

Fig. 1. Configurations in the NaSch model. (The number at the top denotes the velocity).

the Velocity-Dependent-Randomization (VDR) model [13]. And a number of them are modified to adapt to new researches.

A. The Nagel-Schreckenberg Model

The Nagel-Schreckenberg (NaSch) model is a discrete-time and discrete-space model for simulations of traffic networks [14]. In the NaSch model, a road is divided into cells. Each cell is either empty or contains a single vehicle, *i.e.*, no more than one vehicle can occupy a cell at any time. Each vehicle is assigned a velocity, which is an integer between 0 and its maximum velocity. Time is divided into discrete timeslots. In each timeslot, the following four actions (Step 1, 2, 3 and 4) are conducted in order, and during each action the updates are applied to all the vehicles in parallel. Fig. 1 shows the movement of vehicles using the NaSch model, where v denotes the velocity. Let $v_j(t)$ denote the velocity of the vehicle j at time t, v_{max} denote the maximum velocity.

- Step 1: Acceleration.
$$v_j(t+\frac{1}{3}) = min(v_j(t)+1, v_{max}) \quad (1)$$
If the vehicle j does not travel with the maximum velocity, its velocity increases by one cell per timeslot. Otherwise, the velocity remains unchanged. In this paper, the maximum velocity is one cell per timeslot.
- Step 2: Slowing down.
$$v_j(t+\frac{2}{3}) = min(d_j(t), v_j(t+\frac{1}{3})) \quad (2)$$
where d_j denotes the number of empty cells between the vehicle j and the vehicle ahead in the same lane. If $d_j < v_j$, the velocity is reduced to d_j to avoid collision.
- Step 3: Randomization.
$$v_j(t+1) = max(v_j(t+\frac{2}{3})-1, 0) \quad (3)$$
This action occurs with the probability of p'. For the vehicle j which has a velocity of at least one cell per timeslot, the velocity is reduced by one cell with the probability of p'. If the velocity of vehicle j is 0, it remains unchanged. In this paper p' is set 0.
- Step 4: Vehicle motion.
The number of cells the vehicle j should move forward is $v_j(t+1)$. All the vehicles move forward the number of cells which is equal to their velocity.

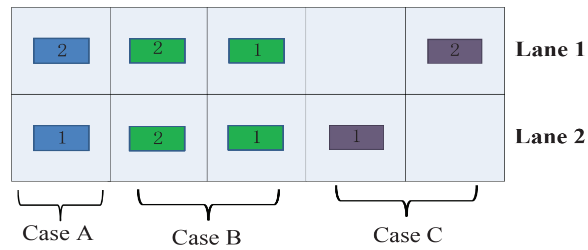

Fig. 2. Case A, B and C of VSR.

B. Assumptions

This research presents a problem in such a scenario: In a road with two lanes, plenty of vehicles have their target lanes. There are sufficient time and space for the vehicles to perform lane change. Each cell of the road has two states: occupied or not occupied by a vehicle (at most one vehicle). The maximum velocity of vehicles is one cell per timeslot, *i.e.*, the velocity of each vehicle is 0 or 1. So each vehicle has the following states: braking, moving forward longitudinally and moving laterally. The space between two neighboring vehicles is equal to or greater than MSS such that collisions are avoided. When vehicles perform lane change, assume the lateral operation costs no time, *i.e.*, the vehicles are able to change to their adjacent lanes immediately without consuming any time. All the vehicles are equipped with wireless communication devices (*e.g.*, WiFi, 3G, 4G), such that they can communicate with one another. To reduce the cost of communication, assume each vehicle broadcasts its information (*e.g.*, velocity, position) at any time. Thus, each vehicle can get the information of others immediately. And they can perform lane change in coordination.

IV. An Optimal Operation of Lane Change in One Row in a Discrete Road

In this section, we analyze three cases of lane change in one row in a discrete road, and propose the Optimal Operation in one Row (OOR) for each case.

With respect to the discrete road, the row indicates one row of cells which is vertical to the direction of this road. The column indicates the cells of one lane which is in the same direction of this road. In the next timeslot, if the vehicle brakes, we write “Brake” at the top. If the vehicle moves forward, we write nothing at the top. As to any row, if the vehicles require lane change, there are three cases of Vehicle States in one Row (VSR):

- Case A
There are two vehicles in the row and they both require lane change, just as depicted in Fig. 2 (the blue vehicles). There are two operations to perform lane change. One operation is f_{A1}: The vehicle in Lane 1 brakes, and the other vehicle moves forward. Then they move laterally to their target lanes, which is shown in Fig. 3. The other operation is f_{A2}: The vehicle in Lane 2 brakes, and the other vehicle moves forward. Then they move laterally to their target lanes, which is shown in Fig. 4.

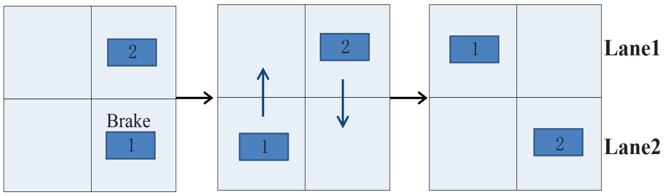

Fig. 3. f_{A1}: One operation of Case A.

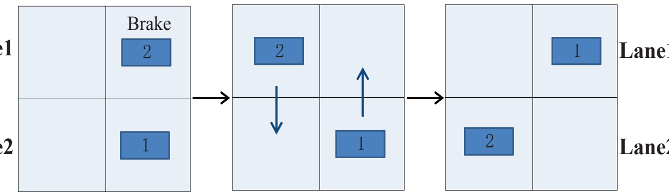

Fig. 4. f_{A2}: The other operation of Case A.

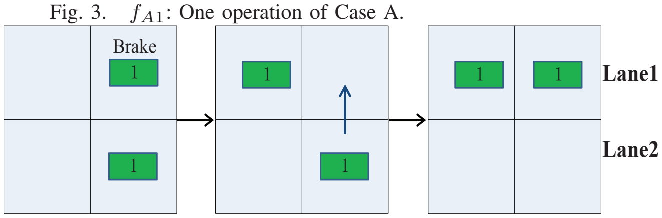

Fig. 5. f_B: One operation of Case B.

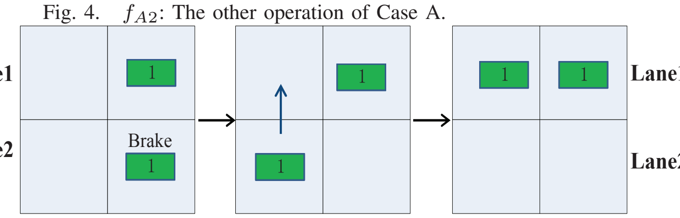

Fig. 6. The other operation of Case B.

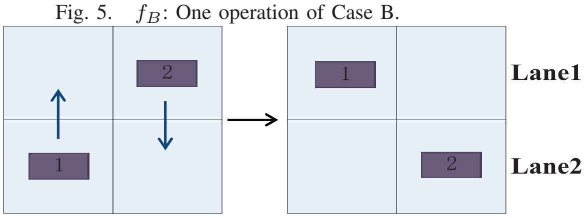

Fig. 7. f_C: The operation of Case C.

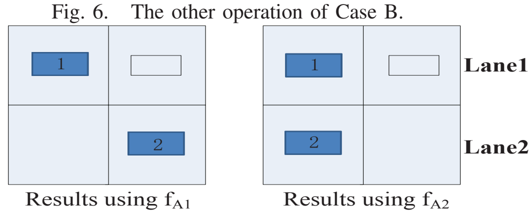

Fig. 8. Results of Case A using two operations.

- Case B
  There are two vehicles in the row and only one of them requires lane change, just as depicted in Fig. 2 (the green vehicles).
  The operation of the two vehicles with target Lane 1 is almost the same as the vehicles with target Lane 2. Now we only consider one case, *e.g.*, the vehicles with target Lane 1. There are two operations to perform lane change. One operation is shown in Fig. 5: The vehicle in target lane brakes, and the other vehicle moves forward, and then it moves to its target lane. The other operation is shown in Fig. 6: The vehicle not in target lane brakes, and the other vehicle moves forward, and then it moves to its target lane. However, the results are different. If we use the second operation, the vehicles in the two lanes have to brake. If we use the first operation, the vehicles in one lane have to brake. Therefore, the first operation is optimal, which is denoted by f_B.
- Case C
  There is only one vehicle in the row and it requires lane change, which is shown in Fig. 2 (the purple vehicles). The operation for this case is f_C: the vehicles directly move to their target lanes, which is shown in Fig. 7.

V. Three Algorithms of Lane Change

Now that we have obtained the optimal operation for each lane change case in one row in a discrete road, while how to perform lane change for all the vehicles? There are three algorithms based on different orders of lane change:
(1) Tail to Head Lane Change (THLC).
(2) Head to Tail Lane Change (HTLC).
(3) Random Lane Change (RLC).

A. Algorithm of THLC

With respect to Case A of VSR, the results of the two operations f_{A1} and f_{A2} are almost the same. Because at last, the numbers of cells the vehicles move forward are the same. However, the mobility of the vehicles ahead should be taken into account: If the vehicles ahead brake, which operation is better? In Fig. 8, one vehicle (the white one) in Lane 1 brakes. Clearly, if f_{A1} is selected, the green vehicle in Lane 1 does not require to brake. While if f_{A2} is selected, the green vehicle in Lane 1 has to brake. For this case, f_{A1} is better than f_{A2}. How to select one operation for Case A is determined by the mobility of the vehicles ahead.

The pseudocode of THLC is shown in Algorithm 1. In this algorithm, the road grid is an $N \times 2$ array, which indicates a discrete road (N rows, 2 columns), and there are empty cells such that all the vehicles have sufficient space to perform lane change. The complexity is $O(N)$.

Lines 2$\sim$11 perform lane change for different cases of VSR using OOR, and the order is from the tail to the head. With respect to Case A of VSR, which vehicle should brake is not deterministic. Therefore, the two vehicles are labeled as a pair of uncertain vehicles. For all the uncertain vehicles, lines 13$\sim$25 determine which vehicles should brake, and the order is from the head to the tail. Because it is determined by the mobility of the vehicles ahead. At last, the lane change for all the vehicles is finished.

Algorithm 1 Algorithm of THLC

Input:

I : An array of $N \times 2$, which denotes the initial states of vehicles, and the number in I denotes the target lane of the vehicle.

Output:

L: The operation of each row of I, and the order is from the head to the tail: $N \to (N-1) \to \ldots \to 2 \to 1$.

$I_{tmp} \leftarrow I$. // All the following operations are based on the array I_{tmp}.
for $i = N$ to 1 **do**
  if Case A of VSR **then**
    Label the two vehicles as a pair of uncertain vehicles. *i.e.*, which vehicle should brake is not deterministic by now.
    If there are vehicles in the next row, they brake. Otherwise, do nothing.
  else if Case B of VSR **then**
    $L_i \leftarrow f_B$.
  else if Case C of VSR **then**
    $L_i \leftarrow f_C$.
  end if
end for
Calculate the number of times T all the uncertain vehicles brake.
for Each pair of uncertain vehicles from the head to the tail **do**
  Find the initial location j of this pair of vehicles in I, *i.e.*, find the row index.
  if Only one vehicle v_b of this pair brakes, and the other does not. **then**
    if Target lane of v_b is Lane 1 **then**
      $L_j \leftarrow f_{A1}$.
    else
      $L_j \leftarrow f_{A2}$.
    end if
  else if None or both of the two vehicles brake **then**
    $L_j \leftarrow f_{A1}$ *or* f_{A2}.
  end if
  Remove the labels of the two vehicles.
end for

B. Algorithm of HTLC

The operation order of HTLC is from the head to the tail using OOR. The complexity is $O(N)$. With respect to the operation of Case A f_A, HTLC randomly chooses one operation method, *i.e.*, $f_A \leftarrow f_{A1}$ *or* f_{A2}.

Fig. 9 and Fig. 10 provide the results of HTLC and THLC, where, in the next timeslot, if the vehicle would move forward then perform lane change, we label a rotated "L". While HTLC results in a 4-row solution, THLC gives a 3-row solution, which is optimal. In the Section. V-D, we show that THLC is optimal.

C. Algorithm of RLC

THLC and HTLC have fixed orders, while RLC does not have. RLC searches the vehicles which require lane change, and randomly chooses one row to perform lane change using OOR, and then it repeats the above steps until the lane change is completed. The complexity is $O(N^2)$. With respect to f_A, $f_A \leftarrow f_{A1}$ *or* f_{A2}.

D. Algorithm of THLC is Optimal

Lemma V.1. *In a discrete road (denoted by an array E ($N \times 2$), the THLC algorithm is optimal in terms of the additional road space Y (denoted by the number of rows in the discrete road) when the lane change is completed.*

Proof: In the real world, if the vehicle brakes, in the array E, we push the vehicle backward. In the real world, if the vehicle moves with the maximum velocity (in this paper, it is one cell per timeslot), in the array E, we do nothing for it. And the number of cells each vehicle moves forward is saved by one variable.

As the operation for each row OOR described in Section. IV is optimal, *i.e.*, with respect to lane change for all the vehicles, it is locally optimal. If we can prove the operation order of $THLC$ is optimal, we can get $THLC$ is globally optimal. Its order is from the tail to the head. If there exists another optimal algorithm O, which is different from $THLC$, we construct an algorithm O', then we compare the performance of O' and O.

Let X denote the operation order. Assume that the first k of X_O is the same as X_{THLC}. Here, k is arbitrary ($k \geq 0$). And from the $(k+1)^{th}$ position, X_O and X_{THLC} are different. Let $i = N - k$, $X_{THLC} = \{N, N-1, \ldots, i+1, i, \ldots, j, \ldots, 1\} (N > i > j > 1)$, and we get $X_O = \{N, N-1, \ldots, i+1, j, \ldots, l, i, \ldots\}$. Now we construct an algorithm O' based on O: i is inserted before j, and the other is the same as O. *i.e.*, $X_{O'} = \{N, N-1, \ldots, i+1, i, j, \ldots, l, \ldots\}$. When O operates row i and O' operates row l, we compare the vehicles of row i using O' and O. If the vehicles in row i are not pushed backward, $Y_{O'} = Y_O$. If the vehicles in row i are pushed backward, after O operates row i, O has to perform lane change for the vehicles which are pushed backward from row i. Clearly, $Y_{O'} \leq Y_O$. Thus we find an algorithm O', whose performance is not worse than O. It is contradictory to our assumption. Therefore, $THLC$ is optimal. ■

VI. Simulations and Results

In this section, we show the simulations and results of three lane change algorithms, and we analyze the performance of THLC, RLC, and HTLC.

A. Notations and Configurations of Simulations

Table I shows the notations and definitions of the simulations. Table II shows the configurations of the simulations. p indicates the ratio of the vehicles which perform lane change using THLC to the total vehicles which demand lane change. If $p = 0$, vehicles perform lane change only using RLC. If $p = 1$, vehicles perform lane change only using THLC. If

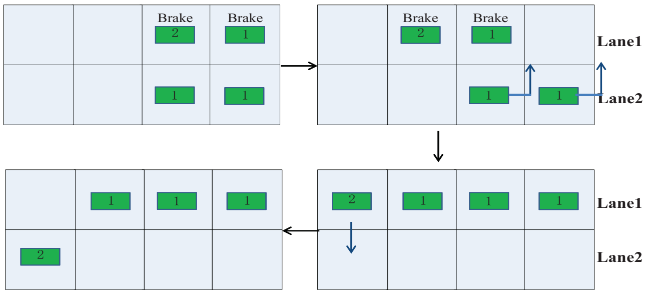

Fig. 9. Results of HTLC.

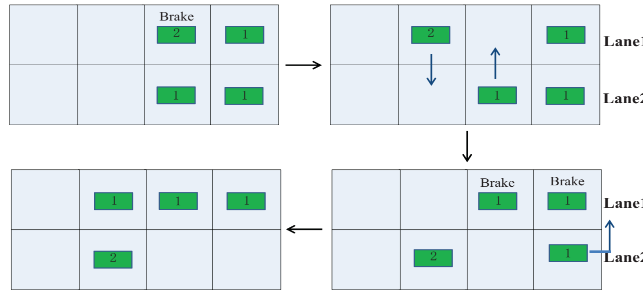

Fig. 10. Results of THLC.

TABLE I
NOTATIONS AND DEFINITIONS OF SIMULATIONS.

Notations	Definitions
m	The number of the computer simulations run.
n_t	The total number of vehicles.
$Density$	The density of vehicles. [0, 1.0]
q	The percentage of the vehicles which require lane change. [0, 1.0]
p	The penetration of THLC among vehicles (see the following for details). [0, 1.0]
ϕ	The additional space till lane change is finished, which is indicated by the number of rows of the discrete road.
ψ	The number of times all the vehicles have to brake.

TABLE II
CONFIGURATIONS OF SIMULATIONS.

Item	Value	Item	Value
q	10% $\sim$ 100%	n_t	50
m	500	$Density$	70% $\sim$ 100%

$0 < p < 1$ vehicles perform lane change using THLC with the probability of p, which implies that the vehicles perform lane change using RLC with the probability of $(1-p)$. ψ refelects the characteristics of velocity: If ψ is high, the average velocity decreases greatly.

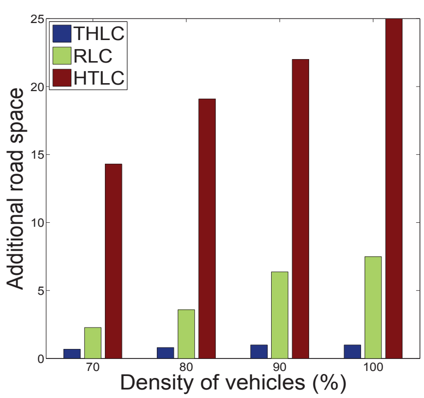

Fig. 11. ϕ: Additional road space using three algorithms ($q = 100\%$).

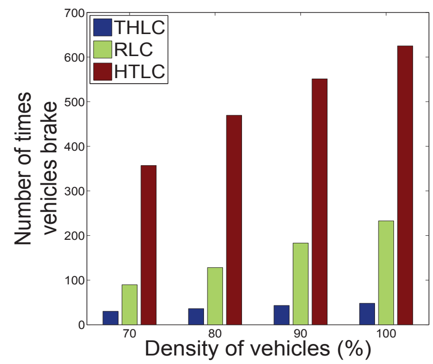

Fig. 12. ψ: Number of times vehicles have to brake for lane change using three algorithms ($q = 100\%$).

B. Results

Fig. 11 shows additional road space using three algorithms. Here, $q = 100\%$. We observe that, the effects of THLC are clear: When $Density$ grows, ϕ grows very slowly. When $Density = 80\%$, the performance of THLC is improved by 77.4% and 95.8% than RLC and HTLC.

Fig. 12 shows the number of times all the vehicles have to brake for lane change using three algorithms. Here, $q = 100\%$. We observe that, THLC has the best performance, and HTLC has the worst. If we use THLC, when $Density$ increases, ψ nearly remains unchanged. When $Density = 80\%$, the performance of THLC is improved by 71.9% and 92.3% than RLC and HTLC.

In Fig. 13, when THLC is used more, ϕ reduces more. It demonstrates that, THLC has affected ϕ during lane change. When the vehicles are sparse, ϕ decreases very slowly. Because the vehicles have enough road space to perform lane change.

From Fig. 14, we observe that, when p increases, ψ decreases, which is clearly when the vehicles are dense. When $Density = 70\%$, ψ decreases very slowly, and the effects of THLC are not as clear as that when $Density = 100\%$.

In Fig. 15, ϕ does not increase monotonously when q increases. If q does not exceed a threshold T_h, ϕ increases as q increases. However, once q exceeds the threshold T_h, ϕ decreases as q increases. *e.g.*, when $p = 100\%$, T_h is about 60%. It can be explained by the following: When $q = 100\%$,

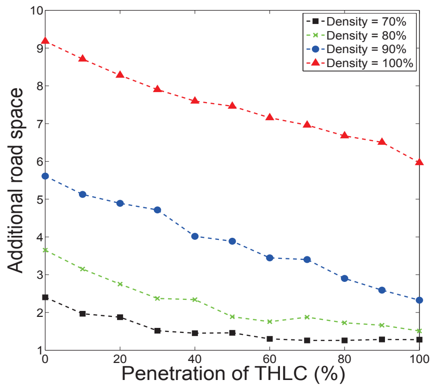

Fig. 13. ϕ: Additional road space when p varies with $Density$ fixed ($q = 70\%$).

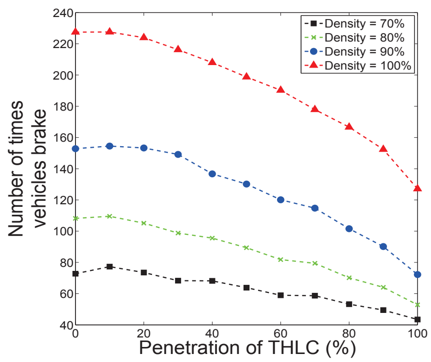

Fig. 14. ψ: Number of times vehicles have to brake when p varies with $Density$ fixed ($q = 70\%$).

each row has only two cases: Case A and Case C, because all the vehicles require lane change. When THLC performs lane change from the tail to the head, the empty cells would be occupied by the vehicles ahead. Thus, the additional road space would not exceed one row.

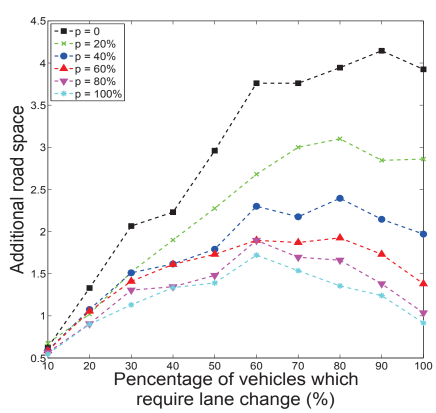

Fig. 15. ϕ: Additional road space when q varies with p fixed ($Density = 80\%$).

VII. Conclusion

In this paper, we proposed a lane change algorithm for two lanes. It is shown that the order of lane change from the tail to the head is better than that from the head to the tail, and it leads to an optimal algorithm. The rule of TH (tail-head) order could be general when more lane change algorithms are developed. The developed algorithm in this paper is a theoretical work. In real situation, we are unable to perform the lane change from the tail to the head. Therefore, new algorithms are to be developed for practical applications. Finally, this work can be extended to multi-lane highways.

VIII. Acknowledgements

This research was supported by NSF of China under grant No. 61073158, No. 61100210, STCSM Project No. 12dz1507400, Doctoral Program Foundation of Institutions of Higher Education under grant No. 20110073120021.

References

[1] M. Quddus, C. Wang, and S. Ison, "Road traffic congestion and crash severity: econometric analysis using ordered response models," *Journal of Transportation Engineering*, Vol. 136, No. 5, pp. 424-435, 2009.
[2] M. Segata, F. Dressler, R. Cigno, and M. Gerla, "A simulation tool for automated platooning in mixed highway scenarios," *ACM SIGMOBILE Mobile Computing and Communications Review*, Vol. 16, No. 4, pp. 46-49, 2013.
[3] H. Hsu and A. Liu, "Kinematic design for platoon-lane-change maneuvers," *IEEE Intelligent Transportation Systems*, Vol. 9, No. 1, pp. 185-190, 2008.
[4] T. Dao, C. Clark, and J. Huissoon, "Distributed platoon assignment and lane selection for traffic flow optimization," *IEEE Intelligent Vehicles Symposium*, pp. 739-744, IEEE, 2008.
[5] Y. Fang, F. Chu, S. Mammar, and M. Zhou, "Optimal lane reservation in transportation network," *IEEE Intelligent Transportation Systems*, Vol. 13, No. 2, pp. 482-491, 2012.
[6] A. Kanaris, E. Kosmatopoulos, and P. Loannou, "Strategies and spacing requirements for lane changing and merging in automated highway systems," *IEEE Vehicular Technology*, Vol. 50, No. 6, pp. 1568-1581, 2001.
[7] J. Hossein, E. Kosmatopoulos, and P. Ioannou, "Collision avoidance analysis for lane changing and merging," *IEEE Vehicular Technology*, Vol. 49, No. 6, pp. 2295-2308, 2000.
[8] X. Zhang and G. Hu, "1/f noise in a two-lane highway traffic model," *Physical Review E*, Vol. 52, No. 5, pp. 4664-4668, 1995.
[9] V. Koverda and V. Skokov, "Critical behavior and 1/f noise in lumped systems undergoing two phase transitions," *Technical Physics*, Vol. 45, No. 10, pp. 1231-1237, 2000.
[10] R. Barlovic, T. Huisinga, A. Schadschneider, and M. Schreckenberg, "Open boundaries in a cellular automaton model for traffic flow with metastable states," *Physical Review E*, Vol. 66, No. 4, p. 046113, 2002.
[11] S. Cheybani, J. Kertsz, and M. Schreckenberg, "Stochastic boundary conditions in the nagel-schreckenberg traffic model," *Traffic and Granular Flow99*, pp. 443-448, Springer, 2000.
[12] H. Zhang, "A finite difference approximation of a non-equilibrium traffic flow model," *Transportation Research Part B: Methodological*, Vol. 35, No. 4, pp. 337-365, 2001.
[13] X.Q. Yang, W. Zhang, K. Qiu, W.t. Xu, G. Tang, and L. Ren, "The relations of go and stop wave to car accidents in a cellular automaton with velocity-dependent randomization," *Physica A: Statistical Mechanics and its Applications*, Vol. 384, No. 2, pp. 589-599, 2007.
[14] N. Moussa and A. Daoudia, "Numerical study of two classes of cellular automaton models for traffic flow on a two-lane roadway," *The European Physical Journal B-Condensed Matter and Complex Systems*, Vol. 31, No. 3, pp. 413-420, 2003.