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Enhancing Throughput in Wireless Multi-Hop Network with Multiple Packet Reception

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***Abstract*—Multi-Packet Reception (MPR) enables simultaneous receptions from different transmitters to a single receiver, which has been demonstrated to bring capacity improvement in wireless network. However, MPR does not improve the transmission capability of intermediate relay nodes in a multi-hop routing and thus these nodes may become the bottlenecks for increasing throughput. We investigate the scheduling for multi-hop routing with MPR to improve the network throughput under multiple data flows. We formulate the optimization problem under K-MPR model and analyze the performance upper bound with ideal scheduling. We propose a distributed scheduling scheme based on a k-Connected k-Dominating Set (k-CDS) backbone to eliminate bottleneck effects.**

I. Introduction

As pointed out in [1], wireless transmissions should respect signal to noise/interference ratio, so as to succeed. The capacity of wireless networks is mainly restrained by the concurrent packet transmissions under collision model. Recently, investigations on the increase of reception capability through multi-user techniques such as SIC [2] and PPS [3], have conducted with notion of MPR. It shifts the responsibility from transmitters to receivers in a wireless communication. A node's MPR capability [4] in a network can be illustrated by a receiver matrix R Eq. (1). $R_{n,k}$, is defined as $Pr[k$ packets received $|n$ packets transmitted $]$. The fundamental change of this model compared to the collision model is that the reception can be described by conditional probabilities instead of deterministic failure when simultaneous transmissions occur.

$$R = \begin{pmatrix} R_{1,0} & R_{1,1} & & & \\ R_{2,0} & R_{2,1} & R_{2,2} & & \\ \vdots & \vdots & \vdots & \ddots & \\ R_{n,0} & \dots & R_{n,k} & \dots & R_{n,n} \end{pmatrix} \quad (1)$$

Mergen and Tong [5] have shown that the upper bound of one-hop throughput in MPR model over conventional collision model increases with the probability of successful reception in MPR. It is easy to understand that for one receiver, the number of receptions can be simply multiple K times by using K-MPR, as long as the interference condition is validated. Recent works [6], [7], [8] have shown that MPR provides a significant capacity improvement for wireless networks, despite using different conditions and models.

MPR can definitively improve the network capacity, but things are different for network throughput. As discussed in [9], the network throughput is influenced by protocols used to date, which are not adapted to fully exploit MPR to increase network throughput. Communication protocols such as MAC protocols have been designed to avoid multiple access interference by preventing multi-access. Similarly, routing protocols for wireless networks do not allow concurrent transmissions in path selection at the same time. Therefore, the MPR capability at physical layer calls for design of new link scheduling schemes matching this capability.

[9] proposed to maximize the number of node-disjoint multi-paths with joint routing and scheduling. But by using node-disjoint paths, at any time slot, each receiver only receives one packet for relaying. The intermediate nodes in routing paths cannot be effective MPR receivers. Fig. 1 gives an example with two data flows (A to D via C, and B to E via C). Ideally, C could benefit from its 2-MPR to receive simultaneously from A and B at slot 1 and uses the next two slots to transmit the received packets to D and E. But with node-disjoint paths, it requires 4 slots to transport two flows. This example also shows that the intermediate nodes in a wireless network might become bottlenecks for throughput. This work is motivated by resolving such bottlenecks with

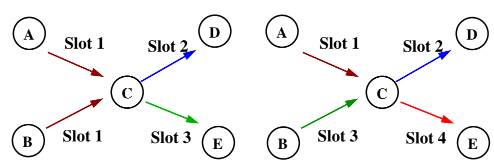

Fig. 1. node-disjoint multi-path is not optimal for MPR

MPR as to enhance the network throughput. The rest of the paper is organized as follows. Section II-C presents our throughput maximizing problem formulation. We investigate its performance upper bound with ideal scheduling in Section III. We introduce a heuristic scheduling scheme based on k-CDS in Section IV. Section V shows the numerical results in random distributed multi-hop wireless networks and we also locate the best value of K to achieve maximal throughput with the heuristic scheme. Section VI concludes this work and discusses future works.

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II. Formulation of Throughput Maximizing Problem

A. Assumptions

We assume that wireless nodes are endowed with a single semi-duplex radio interface, and hence they cannot transmit and receive a packet at the same time. Each nodes is synchronized on time-division slot systems, and the transmissions always take place at the slotted time boundaries.

We define K as the MPR capability of a receiver node. We consider M simultaneous data flows in the network. For each flow m, the set of receiver nodes on the routing paths is $\rho = \{1, 2, ..., p\}^m$. And $\tau = \{1, 2, ..., n\}$ is a set of transmitters ready to transmit. Let $S_v \subseteq \tau$ be a schedulable set of nodes actually simultaneously transmitting to a receiver v.

B. Channel Capacity and Maximal Transmission Rate

For each point-to-point transmission, let P_{iv} be the received power by the receiver v from the transmitter i and P_0 the common transmitted power. The received power with path loss exponent γ is defined as follows:

$$P_{iv} = P_0 d_{iv}^{-\gamma} \quad (2)$$

We consider a multi-user access channel for each wireless communication. The channel capacity function of a single receiver AWGN channel with bandwidth W and channel noise power η can be defined as:

$$\varphi(SINR) = Wlog_2(1 + SINR) \quad (3)$$

The signal to interference plus noise ratio $SINR$ takes into account the channel's noise and the reception signal power of the transmissions other than current reception. For a multi-user access channel with K-MPR, K concurrent transmissions are allowed. The channel capacity for a K-MPR receiver is:

$$\varphi_v = \varphi(\frac{\sum_{i \in S_v} P_{iv}}{\eta}) \quad (4)$$

For a general number of transceivers, the sum of transmission rates are within the channel capacity given in Eq. (4). Therefore, we have the following inequality:

$$\sum_{i \in S_v} r_{iv} \leq \varphi_v(\frac{\sum_{i \in S_v} P_{iv}}{\eta}) \quad (5)$$

where r_{iv} denotes the transmission rate on i to v.

For M data flows, we denote the source and the destination of mth flow s_m and d_m. The flow rate on a directed link (u, v) is denoted as f_{uv}^m. It is worth noting that this flow rate is an average rate and the transmission rate r_{uv}^m could be much higher for an intermediate receiver. For instance, if node v decodes k packets from different transmitters including u and takes t_r slots to relay the received packets, then the relation between flow rate f_{uv}^m and transmission rate r_{uv}^m can be expressed as:

$$\frac{1}{f_{uv}^m} = \frac{t_r + 1}{r_{uv}^m} \quad (6)$$

C. Problem Formulation

Given the above definition and notations, we formulate the Throughput Maximizing Problem (TMP) for multi-flow multi-hop communications as follows:

DEFINITION 1: Maximize the sum of flow rate reaching all destinations:

$$Maximize \sum_{m=1}^{M} \sum_{i} r_{id_m}^m \quad (7)$$

subject to the following three constraints:

1. Flow conservation constraint

$$\sum_i r_{ij}^m = \sum_j r_{ji}^m; \sum_i r_{s_m i}^m = \sum_i r_{id_m}^m \quad (8)$$

2. Receiver constraint

$$\forall v \in \rho, |S_v| \leq K \quad (9)$$

3. Transmitter constraint

$$\sum_{i \in S_v} r_{iv} = \varphi(\frac{\sum_{i \in S_v} P_{iv}}{\eta}) \quad (10)$$

We have adopted the flow conservation constraint Eq. (8) from [9]. However, the other two constraints are different.

Receiver constraint: A receiver v cannot decode more than K packets at the same time, and hence the number of transmitters in S_v should be limited to K for any slot. This constraint covers the receiver's pair-wise interference.

Transmitter constraint: Each transmitter should operate at sum-rate to fully explore the bandwidth of the receiver's multi-user channel, as given in Eq. (4). The bandwidth of such channel is different from that in [9], which is a combination of point-to-point link bandwidth.

III. Throughput Upper Bound with Ideal Scheduling

By resolving the TMP as an optimization problem, we can obtain a performance upper bound. Similar problems have been shown to be NP-hard [9], [10]. The size of our optimization problem increases exponentially with the number of routing paths. Let us more focus on the computation of its upper bound with the ideal time-space scheduling.

Firstly, the wireless network should meet the following necessary condition in our problem:

CONDITION 1: The node degree in the network should be at least $K + 1$ to fully exploit K MPR capability.

Proof: Each receiver should have at least K neighbors to fully use its K MPR capability at the reception slot. During the first time slot, all transmitters in S_v send their flow to receiver v. During the next K slots (from slot 2 to $K + 1$), v is busy sending all the packets it received during the first time slot out to next hop nodes in the routing path. Therefore, it is no longer available for K slots. In order to not waste the time slot, all these transmitters will send their data to the second available receiver in their neighborhood, which will be busy sending all the flows for the next K slots (from slot 3 to slot $K+2$ respectively). If there are only K receivers in the neighborhood, then all of them will be busy relaying the flows they received and time slot $k+1$ will be wasted on this one-hop area. Therefore, a node should have at least $K+1$ neighbors to fully use transmission time slots.

Secondly, the ideal scheduling should meet:

CONDITION 2: The local throughput on each time slot and for each node should be maximized by the ideal scheduling to achieve the maximal flow throughput on the destinations.

Proof: If $\exists t_i$, in which the $\sum_{m=1}^{M}\sum_i r_{iv}^m$ is not maximal, then there are two possible cases. In the first one, another transmission (j,v) can be added to this time slot if receiver v is dealing with less than K transmissions. Or a transmission (j,v) can substitute for an existing transmission (l,v) to achieve a higher throughput. Let $Lt(j)=\sum_{m=1}^{M}\sum_i^{j\in S} r_{iv}^m$, then we have $Lt_v(j)>Lt_v(l)$. By using the flow conservation constraint at all nodes Eq. (8), we obtain the same relation on the the next hop, say w, $Lt_w(j)>Lt_w(l)$. With this recurrent relation, the flow rate (equaling to the transmission rate) at the destination increases when transmission (j,v) occurs.

With the above conditions, the time-space scheduling to achieve the maximal network throughput is the scheduling that maximizes the local throughput on each receiver. We can use a mixed linear programming solver such as [11] to generate numerical results on the upper bound. The comparison to the heuristic scheme is presented in Section V.

IV. Heuristic Scheme with Distributed Scheduling on k-CDS

A. Using k-CDS for scheduling with MPR

In this section, we present a heuristic approach with distributed scheduling based on k-CDS backbone to approximate the upper bound. The k-CDS [12] in a network is a set of nodes which is k-dominating and k-connected. Every node in the network is either in the k-CDS or has k neighbors in it. The subgraph of this node set is k-vertex connected. The properties of k-dominating and k-connected perfectly match for intermediate relay nodes to exploiting MPR capability, because each of them is required to collaborate with at least $K+1$ neighbors for both receptions and transmissions.

- If a receiver is a k-dominated node, then the set consisting of all its k dominating nodes is the schedulable set S.
- If a receiver is a dominating node in k-CDS, then the k connected property guarantees that it is connected to at least k dominating nodes. These nodes can be selected to form schedulable set S for each reception slot.
- If a transmitter is a k-dominated node, then it could schedule with k dominating nodes to transmit.
- If a transmitter is a dominating node, then it could schedule with k dominating neighbors to transmit.

Based on k-CDS backbone, only dominating nodes are selected as intermediate relay nodes for multi-hop routing and dominated nodes do not participate in the routing unless it is the source or the destination of a flow. This simple rule could reduce the complexity of design time-space scheduling in the network.

B. k-CDS construction algorithm

Many algorithms tend to generate a minimal k-CDS, but the transmission will be too concentrated to this set of nodes. On the other side, the high cardinality means few reduction from the original network topology, which is not efficient to reduce the complexity of the scheduling based on the $(K+1)$-CDS. This trade-off on the cardinality of $(K+1)$-CDS can be calculated as follows. Let us assume that the average routing path length is pl. The dominated nodes only participate into the first-hop communications as source nodes or into the last-hop communications as destination nodes, while the dominating nodes can take part into each hop communications in a routing path. By assuming the scheduling has a good fairness for all nodes, the amount of flows that the dominated nodes take is approximately to:

$$T(k-CDS)=\frac{K+2}{(K+1)*(pl-1)} \tag{11}$$

To meet the above constraints, we develop a construction algorithm based on coverage rule [13]. Each node verifies if any pair of its neighbors are k-connected via node-disjoint paths and higher ID's rule is added to avoid mutual decision blocking. This verification is known as k-Coverage condition. To realize this algorithm in distributed and localized manner, nodes exchange their routing tables with their neighbors. The k-Coverage condition is checked via the routing table to count the number of node-disjoint paths from any pair of neighbors.

C. T-R Scheduling for Multi-path Routing

The $(K+1)$-CDS construction algorithm results to a backbone for multi-path routing. We present here a transmitter-receiver scheduling to fully exploit K-MPR capability on dominating and dominated nodes, which allows the use of multiple paths for each flow to eliminate bottlenecks on the intermediate nodes.

A potential transmitter i construct a receivers set ξ_i. The receivers are ordered in each set along with their distances to the final destination d_m in number of hops. If d_m belongs to $N(i)$, the neighbor set of i, then $\{\xi_i\}$ contains only d_m.

With Link Scheduling Algorithm, detailed in Algorithm. 1, a receiver aims to let transmitters operate at sum rate based on $(K+1)$-CDS. It schedules transmitter nodes with the priority pr_i in an arbitrary order. Every node's priority is set to minimal before any transmissions. A transmitter node i is chosen, and checks its possible receivers set ξ_i. If the transmitter finds a receiver v who can receive more flows, then it will be added in the receiver's schedulable set S_v. If the transmitter node cannot find any available receiver, then its priority pr_i will be increased. Hence, during the next time slot, the transmitter i has a higher priority than other transmitters and is to be added to the schedulable set sooner.

For each transmitter allowed to transmit, the algorithm selects the corresponding temporary data-rate, according to the sum-rate constraint. For a schedulable set S_v =

Algorithm 1 Link scheduling

```
while ∃i ∈ τ|b_i^m > 0 do
  choose i with maximum priority
  for v ∈ ξ_i do
    if |S_v| < K then
      S_j = S_j ∪ {i}; r'_i = φ_v(P_i / (η + Σ_{j=0}^{i-1} P_j))
      t_i = T/K; i ++
      break for
    else
      next v in ξ_i
    end if
  end for
  if j = |ρ^m| then
    pr_i ++ ; i ++
  end if
end while
for v ∈ ρ_t do
  for i ∈ S_v do
    r_1 = φ_v(P_1/η) / |S_v|
    if i! = 1 then
      r_i = r'_i + r_1
    end if
    b_i^m(t+1) = b_i^m(t) − t_i × r_i
    if b_i^m(t+1) > 0 then
      pr_i ++
    else
      pr_i = 0
    end if
  end for
end for
```

$\{u_1, u_2, \ldots, u_K\}$, the corresponding data-rates are :

$$r'_1 = \varphi_v(\frac{P_1}{\eta}); r'_K = \varphi_v(\frac{P_K}{\eta + \sum_{j=0}^{K-1} P_j}) \quad (12)$$

The sum of all the data-rates is equal to $\varphi_v(\frac{\sum_{i \in S_v} P_i}{\eta})$. Those data-rates verify the sum-rate constraint, whatever the number of transmitters in schedulable set S_v. Link Scheduling algorithm allows the transmitter i an amount of time $t_i = \frac{T}{K}$, where T is the time slot duration. This ensures that all transmitters will have the same time slot fraction to send their data. Since the first temporary data rate is much higher than the others, the channel utilization need to be re-spread to the selected transmitters in order to achieve fairness and avoid generating bottlenecks on the low rate transmitters. As a result, the overall throughput can be improved.

The final data rates also verify the sum-rate constraint. Let $b_i^m(t)$ be the transmitter i's initial amount of data to send during time slot t for the flow m. The amount of effectively transmitted data is $t_i \times r_i$ and hence the remained amount of data to transmit for the flow m can be represented as $b_i^m(t+1) = b_i^m(t) - t_i \times r_i$. The transmitter i's priority pr_i will be increased, if $b_i^m(t+1)$ is not equal to 0.

V. Performance Evaluation

A. Parameters and Topology Configuration

We set the channel bandwidth $W = 1$ MHz, transmission power $P_0 = 1W$ and path loss exponent $\gamma = 3$. In a square of 300*300, 50 transmitters are randomly generated. According to low noise SNR condition ($SNR_{ref} = 10$dB), $SNR_{ref} = \frac{P_0 d_{ref}^{-\gamma}}{\eta}$, and the maximal distance between two nodes $d_ref = 44$m, we can obtain that η is equal to $1, 16.10^{-6}$W.

The numerical results on upper bound of TMP is obtained through lpslove [11], a mixed linear programming solver. We simulate our heuristic based $(K+1)$-CDS and heuristic based on node-disjoint path [9] in NetLogo4.1 simulator [14]. We performed 100 simulations with a duration of 2000 time slots. For each flow injected in the network, it has fixed source and destination. And it generates one packet per time slot in a saturation condition.

The used metrics are follows: the throughput represents the number of flows arrived to destination, during a predefined number of time slots; the average delay represents the difference between the moment the flow was sent and the moment it is received; and the average acceptance ratio is the ratio of traffic acceptation among the total traffic demand.

B. Results

The overall throughput results obtained are shown in Fig. 2 with a 3-D representation. The throughput upper bound describes the maximal amount of occupied reception time slots at all destinations, which is independent from the number of flows. However, it is shown that it increases with MPR capability. Our heuristic based on $(K+1)$-CDS out-performs the heuristic with node-disjoint path on almost all simulation settings. The node-disjoint heuristic reaches the limit very quickly with the increase of the number of flows, because node-disjoint paths are fewer than the routing paths on k-CDS. Our heuristic has a higher throughput limit, despite that it decreases when the number of flows is important (15 flows).

We can also note that there is a local highest throughput regarding to K-MPR. The throughput of 3-MPR is highest. It is also confirmed in Fig. 3. This is a very interesting observation that the throughput decreases when K becomes bigger with both heuristics. One possible explanation is that the 4-MPR capability requires a much higher density to be fully exploited. The increase of node degree results in that more links interfere with each other, which could decrease the network throughput. For our heuristic, the decrease on throughput with 4-MPR is also related to our link scheduling algorithm, particularly the way we spread the receiver's channel capacity between its transmitters. Indeed, the increase of channel capacity is not very large with MPR capability, while the amount of data to send is much higher.

The results on the heuristics' efficiency to the upper bound in Fig. 4, show that the 4-MPR has the smallest efficiency. This is because the throughput upper bound computed with our problem continues to increase, even if the increase is slower. Nevertheless the efficiency of our heuristic is better than node-disjoint scheme under the same configuration.

Fig. 5 shows that the average delay of a flow increases with the number of flows. Despite of that, using MPR can reduce around 20% of the flow delay compared to single reception

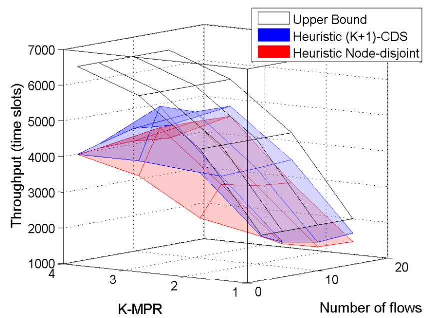

Fig. 2. The throughput of upper bound and heuristics

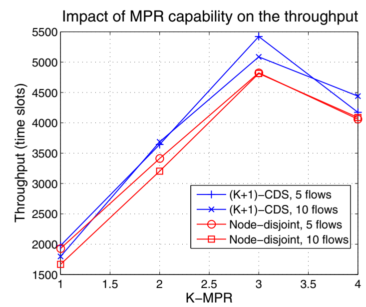

Fig. 3. The 3-MPR can achieve the highest throughput.

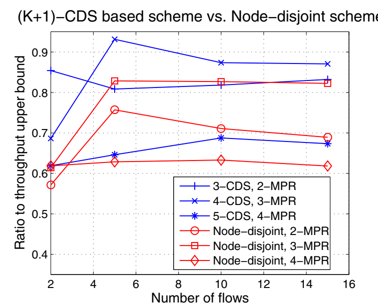

Fig. 4. The heuristics' efficiency, subject to upper bound

model. The increase of delay also confirms the presence of bottlenecks; which also cause the degradation of the flow acceptance ratio as indicated in Fig. 6. Again, MPR could improve the acceptance ratio by using time-space scheduling to avoid bottleneck generation.

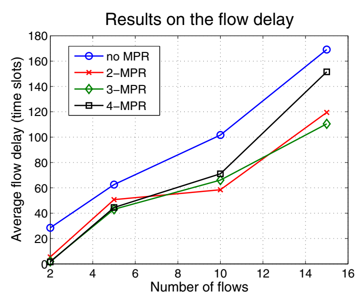

Fig. 5. The delay increases with the number of flows.

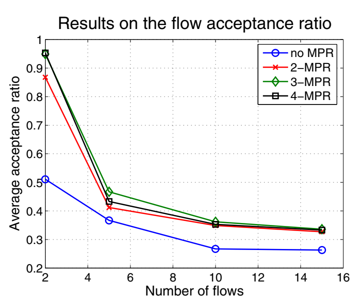

Fig. 6. The flow acceptance ratio of K-MPR chutes.

VI. Conclusion

In this paper, we formulated a maximal throughput problem for multi-hop wireless communications. We point out that the re-use of intermediate nodes in different paths could gain a better performance than node-disjoint approach. Based on some prior works, we formulated an optimization problem subjected to flow, receiver and transmitter constraints. The conditions for ideal scheduling are derived. The numerical results demonstrate that our heuristic scheme based on $(K+1)$-CDS can better exploit the MPR for multi-hop wireless and approximate the upper bound. And for a given topology, we note that there is a optimal value of K for K-MPR for throughput enhancement.

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