How Multiple Crowdsourcers Compete for Smartphone Contributions?

Jia Peng*, Yanmin Zhu*, Wei Shu*†, Min-You Wu*
*Shanghai Jiao Tong University
†University of New Mexico
{pengjia2049, yzhu, mwu}@sjtu.edu.cn, shu@ece.unm.edu

***Abstract*—Mobile crowdsourcing provides us various exciting and profitable applications. All the mobile crowdsourcing systems purchase sensing services from a population of smartphone users. The issue of competition arises. However, few studies address this issue, which significantly affects the functionality of crowdsourcing systems. In this paper, we study the problem of price competition among multiple crowdsourcers in a free market, where smartphone users are able to withdraw from the current crowdsourcer if they can gain more utility from other crowdsourcers. We formulate the price competition as a dynamic non-cooperative game, where each crowdsourcer independently decides its own price aiming at highest profit and all smartphone users join the crowdsourcers at their satisfaction. In practice, a crowdsourcer may not know strategies of others which are unrevealed information. We propose a distributed learning algorithm that every crowdsourcer learns from its historic information to achieve the Nash equilibrium in the market. We studied and identified the suitable learning speed to make the price adaption converge to the Nash equilibrium. All the results are supported by both theoretical analysis and simulations.**

***Index Terms*—Mobile Crowdsourcing, Multiple Crowdsourcers, Price Competition**

I. Introduction

Nowadays, the exponential growth of smartphones creates a compelling paradigm of mobile crowdsourcing [1]. All the smartphone sensors show a sensor network off-the-shelf [2]. A variety of crowdsourcing applications [3]–[5] have emerged and undoubtedly revolutionized many sectors of our life.

A single mobile *crowdsourcer* (*i.e.*, a crowdsourcing system) typically consists of two parts: a crowdsourcing platform residing on the cloud and a population of smartphone users. The crowdsourcer purchases sensing services from smartphone users, who consume their own resources to accomplish the sensing task. Those smartphone users will not work for a crowdsourcer, unless compensation of their resource consumption is satisfied. Without contribution of a *plenty* of smartphone users, a crowdsourcer is not able to collect enough data to accomplish its task.

In a mobile crowdsourcing market, there are many *crowdsourcers* and a population of *smartphone users*. We focus on the price competition of multiple crowdsourcers in this paper. Smartphone users have freedom to choose the crowdsourcers to serve. Therefore, each crowdsourcer in the market has to set a appropriate price to attract sufficient participation. The prices of the crowdsourcers jointly influence the willingness of smartphones to serve. The issue of *competition* arises. The price can not be very high since the profit augment may not compensate the cost growth. Therefore, an wise *pricing* strategy is needed for every crowdsourcer.

The study on the pricing for multiple crowdsourcers faces several challenges. It is not easy to characterize behaviors of smartphone users who may change the crowdsourcer to serve. All the participants in the market should be satisfied, including the selfish crowdsoucers maximizing individual profit and smartphone users pursing the highest utility when making sensing decisions. Only when the price competition ends up with a stable state, will all the participants be pleased. It is challenging to fulfill all the constraints mentioned above, especially when the market information is incompletely unrevealed.

Not much research effort has been done in mobile crowdsourcing when bringing in the competition of multiple crowdsourcers. Some research effort aimed at exploiting the abundant computational power on mobile devices and presented a practical participatory computing framework [6]. Many studies assumed a single crowdsourcer [7], [8]. A category of studies only stood *on a crowdsourcer's side* [9], [10], limiting the initiative of smartphone users, which was not realistic. Luo *et al.* [9] designed a mechanism in the spirit of all-pay auctions, where the bid losers (*i.e.* smartphone users) still had to pay their bids. Some work stood *on the smartphone users' side* [11]. Since there was only one crowdsourcer to serve, smartphone users had no choice on the crowdsourcers. If they were not content with the particular crowdsourcer, they just dropped out the system. These unsatisfied smartphone users could be motivated to join other crowdsoucers.

Although the competition among multiple systems has been studied for other network problems, such as spectrum trading [12], [13], network providers competition [14], [15], and other resources allocation problems [16], [17], it was always the case that these platforms (*e.g.*, network providers) sold network resources to a number of consumers. In mobile crowdsourcing, the systems (*i.e.*, crowdsourcers) relied on smartphone users to provide sensing service. It is necessary to analyze this different situation. Here we have concluded the related work and will not spare an individual section for it.

The economics is playing an more and more important role in allocating network resources [18]–[20]. We view the price competition as an oligopoly market [21] and apply a dynamic non-cooperative game, which is based on Bertrand game [22] to analyze the oligopoly market. There are multiple sellers and a group of buyers in a Bertrand game. The *sellers* compete with each other to sell goods or services to the buyers. Every seller goes after its highest profit and the buyers intend to the well-content utility. Both the sellers and the buyers influence the demand of goods or services. *Different from the stand Bertrand game model*, it is multiple *buyers* (*i.e.*, crowdsourcers) that compete with each other to purchase sensing service from a population of sellers (*i.e.*, smartphone users) in a mobile crowdsourcing market.

We highlight the main intellectual contributions as follows.

- We analyze the phenomenon of price competition in mobile crowdsourcing systems, where multiple crowdsourcers compete with each other to purchase crowdsourcing service from smartphone users. We formulate this as a dynamic non-cooperative game model and Nash equilibrium is the solution of the game.
- In practice, a crowdsourcer may not know the strategies of others which are unrevealed information. We propose a distributed learning algorithm that every crowdsourcer learns from its historic information to achieve the Nash equilibrium in the market. We studied and identified the proper learning speed that to make the price adaption converge to the Nash equilibrium.

The remainder of the paper is organized as follows. In Section II, we present the system model and define the problem. Section III describes the solution of price competition. Section IV presents and discusses the evaluation results. Section V concludes the paper and shows the future work directions.

II. System Model and Problem Formulation

In this section, we present the market model of multiple crowdsourcers and formulate the problem considered in the paper.

A. Free Market Model

As depicted in Fig. 1, a mobile *crowdsourcing market* consists of multiple *crowdsourcers* and a population of *smarphone uers*. We view the population of smartphone users as a continuum of service providers of the same type, which can also be called a representative service provider.

The total number of crowdsourcers is represented by N ($N \geq 2$). When a crowdsourcer i announces its task and price p_i per unit time, smartphone users will decide whether to participate in the task and how much time they will contribute. We adopt the amount of time units to evaluate the quantity of sensing service a crowdsourcer attracts. A smartphone user can only join one crowdsourcer at a time. Smartphone users may join a particular crowdsourcer at their satisfaction of utility obtained. The crowdsourcers must be strategic to set different prices to attract smartphone users and achieve an ideal profit at the same time.

Here we define the contribution that smartphone users devote to crowdsourcers as the *service supply*. Throughout the paper, we use the notion of *service supply substitutability* to express the willingness of a smartphone users to change the crowdsourcer to serve. This is inspired by the concept of substitute good [23] in economics. A substitute good is a good with a positive cross elasticity of demand [23], which means a good's demand is increased when the price of another good is increased. In our case, the willingness of smartphone users to join another crowdsourcer is increased when the payment offered by another crowdsourcer is increased.

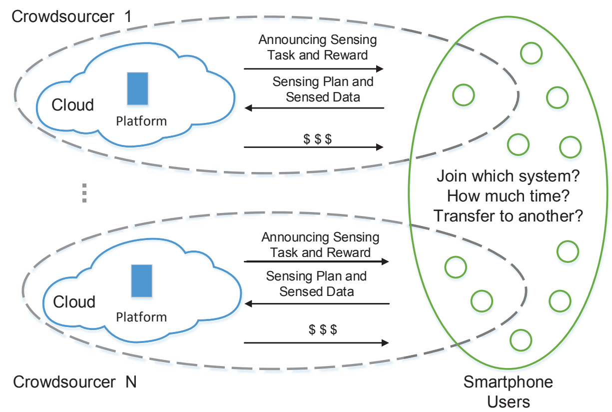

Fig. 1. An example of a mobile crowdsourcing market.

B. Utility of Smartphone Users

The smartphone users make sensing plans aiming at maximizing utility in response to the price profile of all crowdsourcers, which in return affects the price adjustment of crowdsourcers in the market. We focus on the service supply share and price adaption process of crowdsourcers in this paper.

The utility of smartphone users is influenced by the reward of selling sensing service and the cost of doing sensing tasks. The utility will not increase all the time and one simple reason is that a sensing task with a higher reward often costs more than a lower one. Thus, the utility of smartphone users will be saturated in some degree. We extend the linear demand model characterizing a differentiated duopoly proposed in [22].

The utility function of smartphone users is:

$$U(\mathbf{a}) = \sum_{i=1}^{N} a_i p_i - \frac{1}{2}\left(\sum_{i=1}^{N} a_i^2 + 2v \sum_{i \neq j} a_i a_j\right) - \sum_{i=1}^{N} a_i k_i, \quad (1)$$

where a_i is the amount of time units that the representative smartphone user is willing to sell to crowdsourcer i. $\mathbf{a} = \{a_1, ..., a_i, ..., a_N\}$ stands for the set of service supply to all crowdsourcers in the market. p_i is the price per unit time of crowdsourcer i paid to smartphone users. k_i is the cost per unit time of smartphone users doing sensing task for crowdsourcer i. $v \in [0.0, 1.0]$ denotes the *service supply substitutability*. When $v = 0.0$, smartphone users are not willing to change the crowdsourcers they serve, while for $v = 1.0$, smartphone users change the crowdsourcers to serve frequently.

C. Profit of Crowdsourcer

Every crowdsourcer sets price selfishly to obtain the maximum profit. The revenue of a crowdsourcer is the services gained from smartphones' sensing and the cost is the reward paid to smartphone users. Then the profit function of crowdsourcer i is:

$$P_i(\mathbf{p}) = R_i(a_i) - C_i(a_i), \tag{2}$$

where $R_i(a_i) = ca_i, C_i(a_i) = p_i a_i$ and $c > 1$ is system parameter, $0 < p_i < c$, a_i reflects the earnings of crowdsourcer i on participating users.

Here we do not consider the redundancy of sensory data. In a competitive market, every crowdsourcer has to be wise enough to attract sufficient smartphone participation. We focus on the competition among crowdsourcers in this paper.

III. Price Competition and Solution

In this section, we formulate the pricing competition as a dynamic non-cooperative game based on the standard Bertrand game and Nash equilibrium is the solution of the game.

A. Oligopoly Price Competition and Bertrand Game

We observe that the market of crowdsourcers is an oligopoly market [21]. In economics, oligopoly is a situation where a small number of *sellers* (*i.e.*, oligopolists) dominate a market. Each of them makes decisions pursuing the highest profit. Two classical models in the theory of oligopoly are Cournot and Bertrand [24] game model. In a Cournot game, sellers set quantities. In a Bertrand game, prices are the strategy variables. According to [22], Bertrand (Cournot) competition with substitutes is the dual of Cournot (Bertrand) competition with complements. Exchanging prices and quantities, we go from one to the other. Further, with a linear demand structure Bertrand competition is more efficient than Cournot competition in consumer or total surplus terms. Thus, the idea of Bertrand game for price competition is appropriate to analyze the market of multiple crowdsourcers.

B. Basic Settings of Crowdsourcers: A Non-cooperative Game Model

We model the price competition based on a Bertrand game, which we call the *Pricing game*. Different from the stand Bertrand game, it is *multiple buyers* instead of *multiple sellers* that are strategic players in the game. The **players** in the Pricing game are crowdsourcers. The **strategy** of each player is the price per unit time of sensing service (denoted by $p_i, 0 < p_i < c$). The **payoff** of each crowdsourcer i (denoted by P_i) is the profit gained by smartphone users' contributions.

We are interested in answering the following questions: How will a crowdsourcer set the value of p_i to maximize its profit? Will the price adaption end in a stable state? To answer the former question, a crowdsourcer needs to know the amount of time (*i.e.*, service supply) that smartphone users will contribute in response to the strategies of all the crowdsoucers.

The latter question corresponds to the notion of *Nash equilibrium* [25] in game theory, which is a strategy profile that no player has anything to gain by unilaterally changing its current strategy. Another concept called *best response* [25] is central to the Nash equilibrium. The best response is the strategy that produces the most favorable outcome for a player, given strategies of other players. The Nash equilibrium is the point at which each player in a game has selected the best response (or one of the best responses) to the strategies of others.

Let $\mathbf{p} = \{p_1, ..., p_i, ..., p_N\}$ stands for the strategy profile consisting of strategies of all players. Let $\mathbf{p_{-i}}$ denotes the strategy profile excluding p_i. As a notational convention, we write $\mathbf{p} = (p_i, \mathbf{p_{-i}})$. Let $B_i(\mathbf{p_{-i}})$ represents the best response of crowdsourcier i.

C. Smartphone Users: Which Crowdsourcer to Join? How Much Time? Transfer to Another?

Based on the strategy profile of all crowdsourcers, smartphone users will make sensing plan (*i.e.*, the time they will devote to every crowdsourcer), aiming at the highest utility. We can obtain the sensing plan from the utility function of smartphone users in Eq.(1). We differentiate $U(\mathbf{a})$ with respect to a_i and set it to 0, meaning that how to set the value of a_i to maximize utility $U(\mathbf{a})$.

$$\frac{\partial U(\mathbf{a})}{\partial a_i} = 0 = p_i - a_i - v \sum_{i \neq j} a_j - k_i. \tag{3}$$

By solving the set of equations in Eq.(3), we can obtain every element of $\mathbf{a} = \{a_1, ..., a_i, ..., a_N\}$, the sensing plan of smartphone users. Here we replace a_i by $W_i(\mathbf{p})$.

$$W_i(\mathbf{p}) = \frac{(p_i - k_i)[1 + v(N-2)] - v \sum\limits_{i \neq j} (p_j - k_j)}{(1 - v)[1 + v(N-1)]}. \tag{4}$$

For simplification, we rewrite the sensing plan function in Eq.(4) as $W_i(\mathbf{p}) = \mathrm{D}_2 p_i - D_1(\mathbf{p_{-i}})$, where $D_1(\mathbf{p_{-i}})$ and D_2 are constants given all p_j for $i \neq j$ and can be expressed as follows:

$$D_2 = \frac{1 + v(N-2)}{(1 - v)[1 + v(N-1)]}, \tag{5}$$

$$D_1(\mathbf{p_{-i}}) = \frac{k_i[1 + v(N-2)] + v \sum\limits_{i \neq j} (p_j - k_j)}{(1 - v)[1 + v(N-1)]}. \tag{6}$$

Note that the sensing plan of smartphone users is the service supply share of all crowdsourcers.

D. Strategies of Crowdsourcers in Single Stage: Nash Equilibrium

We will prove that the best response of each player is unique. Thus, the Nash equilibrium of the Pricing game is unique.

Theorem 1. *The best response $B_i(\mathbf{p_{-i}})$ of crowdsourcier i is unique.*

Proof. Given the service supply $W_i(\mathbf{p})$, the profit of crowdsourcer is:

$$P_i(\mathbf{p}) = (c - p_i) W_i(\mathbf{p}). \tag{7}$$

To study the best response strategy of crowdsourcer i, we compute the derivatives of P_i with respect to p_i:

$$\frac{\partial P_i(\mathbf{p})}{\partial p_i} = -2D_2 p_i + D_1(\mathbf{p_{-i}}) + \mathrm{c}D_2, \tag{8}$$

$$\frac{\partial^2 P_i(\mathbf{p})}{\partial p_i^2} = -2D_2. \tag{9}$$

Since the second-order derivative of P_i is negative, the utility P_i is a strictly concave function in p_i. Therefore given any strategy profile $\mathbf{p_{-i}}$ of the other crowdsourcer, the best response strategy $B_i(\mathbf{p_{-i}})$ of crowdsourcer i is unique. □

Best response of crowdsourcers will be obtained as following. Setting the first derivative of P_i to 0, we have

$$-2D_2 p_i + D_1(\mathbf{p_{-i}}) + \mathrm{c}D_2 = 0. \tag{10}$$

Solving for p_i in Eq.(10), we obtain

$$p_i = \frac{D_1(\mathbf{p_{-i}}) + \mathrm{c}D_2}{2D_2}. \tag{11}$$

Combining Eq.(5), Eq.(6) with Eq.(11), we get

$$p_i = \frac{1}{2}\left[\frac{\sum\limits_{j\neq i}(p_j - k_j)}{v(N-2)+1}v + k_i + c\right]. \tag{12}$$

The set of equations in Eq.(12) is the price profile at Nash equilibrium.

E. Crowdsourcers in a Agile Market: A Dynamic Non-cooperative Game Model

In practice, a crowdsourcer may *not know the current strategies* adopted by other crowdsourcers. Only the historic strategies are likely to be observed by others. We also consider the time dynamics of the crowdsoucers' market and extend the aforementioned static model to a dynamic one.

Let $p_i[\mathrm{t}]$ denotes the price offered by crowdsourcer i at iteration t. The sets $\mathbf{p_{-i}}[\mathrm{t}]$ and $\mathbf{p}[\mathrm{t}]$ are defined similarly. Given the strategies adopted by other players at time t (*i.e.*, $\mathbf{p_{-i}}[\mathrm{t}]$), the price offered by crowdsourcer i at time $t+1$ might be:

$$p_i[\mathrm{t}+1] = B_i(\mathbf{p_{-i}}[\mathrm{t}]), \forall i. \tag{13}$$

Each crowdsourcer will adjust its price according to Eq.(13). We will study the convergence of the price in the next subsection.

However, a crowdsourcer may *not know the historic strategies* of others since the information are unrevealed. It can only use the local information and service supply (*i.e.*, $W_i(\mathbf{p})$) from smartphone users to adjust its strategy. There is no doubt that a crowdsourcer knows its service supply at previous iteration. A crowdsourcer will adjust its strategy in the direction that maximizes its profit. That is:

$$p_i[\mathrm{t}+1] = p_i[\mathrm{t}] + \alpha_i\left(\frac{\partial P_i(\mathbf{p})}{\partial p_i}\right), \tag{14}$$

where $0 < \alpha_i < 1$ is the adjustment speed (*i.e.*, learning rate),

$$\begin{aligned}\frac{\partial P_i(p)}{\partial p_i} \approx &\frac{P_i(p_{-i}[\mathrm{t}] \cup \{p_i[\mathrm{t}]+\varepsilon\})}{2\varepsilon}\\ &-\frac{P_i(p_{-i}[\mathrm{t}] \cup \{p_i[\mathrm{t}]-\varepsilon\})}{2\varepsilon},\end{aligned} \tag{15}$$

$$\begin{aligned}P_i(p_{-i}[\mathrm{t}] \cup \{p_i[\mathrm{t}] \pm \varepsilon\}) =& cW_i(p_{-i}[\mathrm{t}] \cup \{p_i[\mathrm{t}] \pm \varepsilon\})\\ &-p_i W_i(p_{-i}[\mathrm{t}] \cup \{p_i[\mathrm{t}] \pm \varepsilon\}).\end{aligned} \tag{16}$$

To estimate the marginal profit, a crowdsourcer is capable of observing the marginal service supply for small variation in price ε (e.g., $\varepsilon = 10^{-4}$) as shown in Eq.(15) and Eq.(16). Solving Eq.(14), Eq.(15) and Eq.(16), we get:

$$\begin{aligned}p_i[t+1] &= p_i[t] + \alpha_i(c - p_i[t])D_2\\ &= [1-\alpha_i D_2]\, p_i[t] + \alpha_i D_2 c.\end{aligned} \tag{17}$$

F. Will the Price Adaptation Converge to a Nash Equilibrium?

We will see whether the price adjustment process will end up with a stable price profile. We observe that both Eq.(13) and Eq.(14) are self-mapping functions. By definition, the self-mapping function is stable if and only if the eigenvalues λ_i of its Jacobian matrix are all inside the unit circle of the complex plane (e.g., $|\lambda_i|$<1).

In our case, the Jacobian matrix is:

$$\mathrm{J} = \begin{pmatrix} \frac{\partial p_1[\mathrm{t}+1]}{\partial p_1[\mathrm{t}]} & \frac{\partial p_1[\mathrm{t}+1]}{\partial p_2[\mathrm{t}]} & \cdots & \frac{\partial p_1[\mathrm{t}+1]}{\partial p_N[\mathrm{t}]} \\ \frac{\partial p_2[\mathrm{t}+1]}{\partial p_1[\mathrm{t}]} & \frac{\partial p_2[\mathrm{t}+1]}{\partial p_2[\mathrm{t}]} & \cdots & \frac{\partial p_2[\mathrm{t}+1]}{\partial p_N[\mathrm{t}]} \\ \vdots & \vdots & & \vdots \\ \frac{\partial p_N[\mathrm{t}+1]}{\partial p_1[\mathrm{t}]} & \frac{\partial p_N[\mathrm{t}+1]}{\partial p_2[\mathrm{t}]} & \cdots & \frac{\partial p_N[\mathrm{t}+1]}{\partial p_N[\mathrm{t}]} \end{pmatrix}. \tag{18}$$

For Eq.(13), the Jacobian matrix is:

$$J = \begin{pmatrix} 0 & \frac{v}{2[v(N-2)+1]} & \cdots & \frac{v}{2[v(N-2)+1]} \\ \frac{v}{2[v(N-2)+1]} & 0 & \cdots & \frac{v}{2[v(N-2)+1]} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{v}{2[v(N-2)+1]} & \frac{v}{2[v(N-2)+1]} & \cdots & 0 \end{pmatrix}. \tag{19}$$

The eigenvalues of the Jacobian matrix in Eq.(19):

$$\lambda_i = -\frac{v}{2\left[v(N-2)+1\right]}, \frac{v}{2\left[v(N-2)+1\right]}, \ldots, \frac{(\mathrm{N}-1)v}{2\left[v(N-2)+1\right]}. \tag{20}$$

For the maximum value of λ_i, we have:

$$\frac{(\mathrm{N}-1)v}{2\left[v(N-2)+1\right]} < 1 \Rightarrow (N-3)v + 2 > 0. \tag{21}$$

When $N=2$, we have $2-v>0$. In summery, $|\lambda_i|$<1. Thus, for the case that the historic strategies of others are observable, the price adaption will be in a steady state.

For Eq.(14), the Jacobian matrix is:

$$J = \begin{pmatrix} 1-\alpha_1 D_2 & 0 & \cdots & 0 \\ 0 & 1-\alpha_2 D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1-\alpha_N D_2 \end{pmatrix}. \tag{22}$$

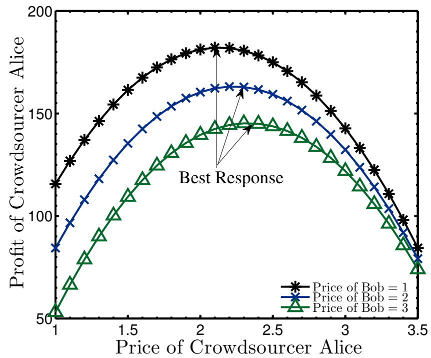

Fig. 2. Best response of crowdsourcer Alice.

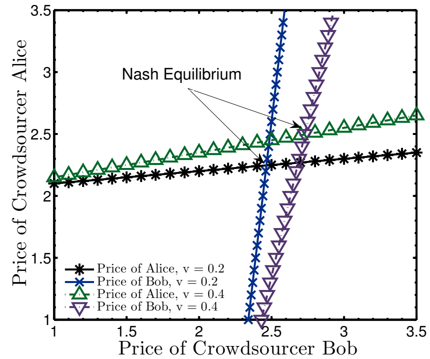

Fig. 3. Nash equilibrium in a single stage.

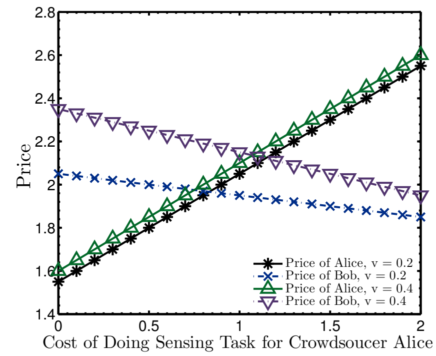

Fig. 4. Equilibrium price under different cost of doing sensing task for crowdsourcer Alice.

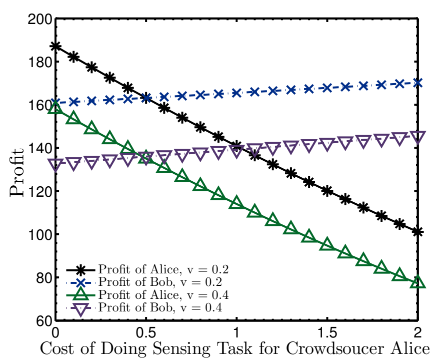

Fig. 5. Equilibrium profit under different cost of doing sensing task for crowdsourcer Alice.

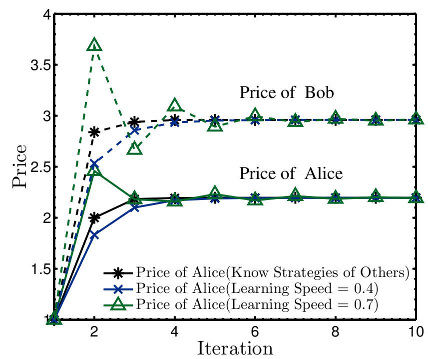

Fig. 6. Convergence to the Nash equilibrium.

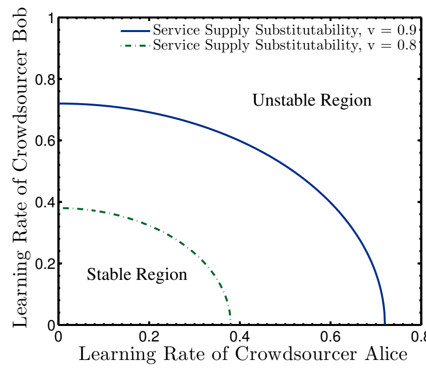

Fig. 7. Stability region under different learning rates.

The eigenvalues of the Jacobian matrix in Eq.(22):

$$\lambda_i = 1 - \alpha_i D_2. \tag{23}$$

The condition that ensures $|\lambda_i|<1$ is $0 < \alpha_i < 1, \frac{2}{D_2} \geq 1$ or $0 < \alpha_i < \frac{2}{D_2}, \frac{2}{D_2} < 1$. We find that $\frac{2}{D_2} \geq 1$ is not always true. In the next section, we will investigate the *stability region* of learning rates and identify the suitable learning rate that makes the price adaption converge to the Nash equilibrium as fast as the former case.

IV. Performance Evaluation

We present in this section a series of experiment simulating a mobile crowdsourcing market with two crowdsourcers, which acts as a powerful support for the theoretical analysis in general case. We name the two crowdsourcers as Alice and Bob respectively.

A. Parameter Settings

The default settings are as follows. The system parameter $c = 4$. The cost of joining Alice and Bob are $k_1 = 0.1$ dollar and $k_2 = 0.5$ dollar respectively. The *service supply substitutability* factor v lies between 0.2 to 0.8. Note that some of these parameters will be varied according to the evaluation scenarios.

B. Best Response of a Crowdsourcer

Fig. 2 shows the profit of Alice as a function of price. Before some certain point, the profit increases since the increasing price attracts more service supply. After the certain point, the profit decreases since the profit augment can not compensate the cost growth. The *best response* is the price which results in the highest profit. The best response price of Alice increases as the price given by Bob increases. This is a good explanation of positive cross elasticity of demand.

C. Nash Equilibrium in a Single Stage

As depicted in Fig. 3, the best response functions of the crowdsourcers intersect at a point, which is the Nash equilibrium. The equilibrium price p_2 is larger than p_1, since the cost for serving crowdsourcer Bob is higher. We get higher prices at Nash equilibrium with a higher *service supply substitutability* value v. Since smartphone users change to serve the crowdsourcer offering higher reward frequently, each crowdsourcer has to increase its price to attract the participation of smartphone users.

D. Effect of Smartphones Cost on Prices and Profits at Nash equilibrium

Fig. 4, 5 show the price and profit of both crowdsourers at the Nash equilibrium, as the cost of doing sensing task for Alice increases. We observe that a larger value of v have different influences on Alice and Bob: the price of Alice is slightly effected, while the price of Bob decreases at a larger rate. We explain this phenomenon as follows: smartphone users will gain more utility from Bob than Alice as k_1 increases and k_2 keeps the same. When v becomes larger, smartphone users change to serve crowdsourcer Bob with higher frequency, which enhances the advantage on the share of service supply for Bob. Thus, Bob can decrease its price at a faster rate and ensure profit augment at the same time.

E. The Learning Algorithm Converging to a Nash Equilibrium

Fig. 6 shows the convergence of price adjustment. When the strategies of crowdsourcers are observable by each other, a fast convergence to the equilibrium price is expected. When a crowdsourcer can observe only the service supply of the smartphone users, it is still able to reach the equilibrium price through our learning algorithm. The speed of convergence largely depends on the learning rate. If this learning rate is properly set (e.g., $\alpha_1 = \alpha_2 = 0.4$), the algorithm converges to the equilibrium price as fast as the case when the strategies of the other players are observable. However, if the learning rate is large (e.g., $\alpha_1 = \alpha_2 = 0.7$), it causes fluctuation in the price adjustment, and the algorithm may require a larger number of iterations to reach the equilibrium. This result is a guidance for every crowdsourcer to adjust its strategy.

F. Stability Region of the Learning Algorithm

When the strategies of other crowdsourcers can not be observed, we analyze the stability region of the learning rates in this subsection. Fig. 7 shows the result. When the learning rates α_1 and α_2 are set with values taken from the stability region, the distributed dynamic algorithm successfully converges to the Nash equilibrium. We observe that the *service supply substitutability* factor v causes a destabilization effect, when it becomes larger.

V. Conclusion and Future Work

In this paper, we have addressed the problem of competitive pricing of multiple crowdsourcers. We use a dynamic non-cooperative game to model this situation and propose a distributed learning algorithm converging to the prices at Nash equilibrium. We will investigate other issues such as sensing task auction and privacy preservation when bringing competition among crowdsourcers in the future.

Acknowledgements

This work was supported by National 863 Program (2013AA01A601), the Natural Science Foundation of China (NSFC) projects (No. 61373155, 91438121, 61100210, 61472254, 61170238 and 61373156), the STCSM Project (No. 12dz1507400 and 13511507800), the Key Basic Research Project (No. 12JC1405400) and the Shanghai Pujiang Program (No. 13PJ1404600) of the Shanghai Municipality.

References

[1] G. Chatzimilioudis, A Konstantinidis, C. Laoudias, and D. Zeinalipour-Yazti. Crowdsourcing with smartphones. *IEEE Internet Computing*, 16(5):36–44, 2012.

[2] Linghe Kong, Mingchen Zhao, Xiao-Yang Liu, Jialiang Lu, Yunhuai Liu, Min-You Wu, and Wei Shu. Surface coverage in sensor networks. *IEEE Transactions on Parallel and Distributed Systems*, 25(1):234–243, 2014.

[3] Kleomenis Katevas, Hamed Haddadi, and Laurissa Tokarchuk. Poster: Sensingkit: A multi-platform mobile sensing framework for large-scale experiments. In *Proceedings of ACM MOBICOM*, 2014.

[4] Moustafa Alzantot and Moustafa Youssef. Crowdinside: Automatic construction of indoor floorplans. In *Proceedings of GIS*, 2012.

[5] Kanak Biscuitwala, Willem Bult, Mathias Lécuyer, T.J. Purtell, Madeline K.B. Ross, Augustin Chaintreau, Chris Haseman, Monica S. Lam, and Susan E. McGregor. Dispatch: Secure, resilient mobile reporting. *SIGCOMM Comput. Commun. Rev.*, 43(4):459–460, 2013.

[6] Zheng Dong, Linghe Kong, Peng Cheng, Liang He, Yu Gu, Lu Fang, Ting Zhu, and Cong Liu. Repc: Reliable and efficient participatory computing for mobile devices. In *Proceedings of IEEE SECON*, 2014.

[7] Fudong Qiu, Fan Wu, and Guihai Chen. Privacy and quality preserving multimedia data aggregation for participatory sensing systems. *IEEE Transactions on Mobile Computing*, PP(99):1–1, 2014.

[8] Luis G.Jaimes, Idalides Vergara-Laurens, and Miguel A.Labrador. A location-based incentive mechanism for participatory sensing systems with budget constraints. In *Proceedings of IEEE PERCOM*, 2012.

[9] Tie Luo, Hwee-Pink Tan, and Lirong Xia. Profit-maximizing incentive for participatory sensing. In *Proceedings of IEEE INFOCOM*, 2014.

[10] I Koutsopoulos. Optimal incentive-driven design of participatory sensing systems. In *Proceedings of IEEE INFOCOM*, 2013.

[11] Dejun Yang, Guoliang Xue, Xi Fang, and Jian Tang. Crowdsourcing to smartphones: Incentive mechanism design for mobile phone sensing. In *Proceedings of ACM MOBICOM*, 2012.

[12] R. Berry, M. Honig, T. Nguyen, V. Subramanian, H. Zhou, and R. Vohra. On the nature of revenue-sharing contracts to incentivize spectrum-sharing. In *Proceedings of IEEE INFOCOM*, 2013.

[13] Emir Kavurmacioglu, Murat Alanyali, and David Starobinski. Demand-invariant price relationships and market outcomes in competitive private commons. *ACM Trans. Internet Technol.*, 14(2-3):15:1–15:30, 2014.

[14] Zhi-Li Zhang, P. Nabipay, A. Odlyzko, and R. Guerin. Interactions, competition and innovation in a service-oriented internet: An economic model. In *Proceedings of IEEE INFOCOM*, 2010.

[15] Richard T. B. Ma. Subsidization competition: Vitalizing the neutral internet. *CoRR*, abs/1406.2516, 2014.

[16] Di Niu, Chen Feng, and Baochun Li. A theory of cloud bandwidth pricing for video-on-demand providers. In *Proceedings of IEEE INFOCOM*, 2012.

[17] Saamer Akhshabi, Lakshmi Anantakrishnan, Ali C. Begen, and Constantine Dovrolis. What happens when http adaptive streaming players compete for bandwidth? In *Proceedings of ACM NOSSDAV*, 2012.

[18] Soumya Sen, Carlee Joe-Wong, Sangtae Ha, and Mung Chiang. A survey of smart data pricing: Past proposals, current plans, and future trends. *ACM Comput. Surv.*, 46(2):15:1–15:37, 2013.

[19] M. Andrews, U. Ozen, M.I. Reiman, and Qiong Wang. Economic models of sponsored content in wireless networks with uncertain demand. In *Proceedings of IEEE INFOCOM*, 2013.

[20] Jeremy Blackburn, Rade Stanojevic, Vijay Erramilli, Adriana Iamnitchi, and Konstantina Papagiannaki. Last call for the buffet: Economics of cellular networks. In *Proceedings of ACM MOBICOM*, 2013.

[21] J.W. Friedman. *Oligopoly and the Theory of Games*. Amsterdam: North Holland, 1977.

[22] Nirvikar Singh and Xavier Vives. Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics*, 15(4):152–154, 1984.

[23] A. Schotter. *Microeconomics : A Modern Approach*. HarperCollins College Publishers, 1994.

[24] Xavier Vives. On the efficiency of bertrand and cournot equilibria with product differentation. *Journal of Economic Theory*, 36(1):166 – 175, 1985.

[25] D. Fudenberg and J. Tirole. *Game theory*. MIT Press, 1991.