Neighbor Discovery Algorithms in Wireless Networks Using Directional Antennas

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***Abstract*—Directional antennas provide great performance improvement for wireless networks, such as increased network capacity and reduced energy consumption. Nonetheless new media access and routing protocols are required to control the directional antenna system. One of the most important protocols is neighbor discovery, which is aiming at setting up links between nodes and their neighbors. In the past few years, a number of algorithms have been proposed for neighbor discovery with directional antennas. However, most of them cannot work efficiently when taking into account the collision case that more than one node exist in one directional beam. For practical considerations, we propose a new neighbor discovery algorithm to overcome this shortcoming. Moreover, we present a novel and practical mathematical model to analyze the performance of neighbor discovery algorithms considering collision effects. Numerical results clearly show our new algorithm always requires less time to discover the whole neighbors than previous ones. To the best of our knowledge, it is the first complete, practical analytical model that incorporates directional neighbor discovery algorithms.**

I. INTRODUCTION

Directional antennas offer many potential advantages for wireless networks such as increased network capacity, extended transmission range and reduced energy consumption. However, these advantages require new protocols and mechanisms at medium access and networking layer. Basic network operations (i.e., neighbor discovery) also become more complicated.

Neighbor discovery in wireless network is the process of finding one-hop neighbors and is a crucial initial step for establishing connections among the nodes [1], [2], [6], [8]. Neighbor discovery is a relatively simpler problem when omnidirectional antennas are used since a simple broadcast can reach all nodes within the transmission range. The problem, however, becomes more challenging when directional antennas are used due to the following reasons: i) The limited radial range of the beamwidth of the directional antenna that covers only a fraction of the azimuth. ii) Neighboring nodes must know when and where to point their directional beams to discover each other [19].

Many neighbor discovery algorithms have been proposed for wireless networks that use directional antennas. The main objective of these algorithms is to discover the neighbors around a node efficiently and store the neighborhood information locally. On one hand, a set of proposed protocols utilize omnidirectional antennas to bootstrap the neighbor discovery process [3-5], [9-14]. However, the differences between the omnidirectional and directional antenna gain patterns could result in different sets of discovered neighbors. On the other hand, in [6-8] and [15-18], pure directional transmission and reception are employed in the process of neighbor discovery. Nevertheless, practical systematic models or numerical analysis are still missing in [6-8] and [17-18]. In [15], J. Park analytically studies synchronous and random sector-time slot assignment strategies using pure directional antennas, but MAC protocols and algorithms are not considered. In another significant work [16], Zhang proposes the 2-way pure random algorithm (PRA) and the 2-way scan based algorithm (SBA). Through analysis of these algorithms, Zhang concludes that if designed properly, these directional algorithms can discover neighbors in less time than those algorithms using omni-directional antennas. However, in [16] Zhang ignores the collision case when two or more neighbors exist in the same reception beam. The collision case cannot be ignored, since it takes place frequently in reality, especially when the beamwidth is not very small (i.e., greater than 5 degree).

In this work, we propose a novel analytical model which takes collision effects into account. Based on the practical model, SBA works efficiently only when the number of neighbors is small and PRA is just the opposite case. Therefore, we introduce a new *Improved Scan-based* algorithm (I-SBA). It does not only increase detection probability when the number of neighbors is small, but also decreases collision probability when the number of neighbors goes large. Numerical results finally show that I-SBA always has the best performance. It allows nodes to discover their neighbors in a significantly smaller amount of time slots. The whole analytical model is also validated by simulation through OPNET.

The remainder of the paper is organized as follows. Section II gives an elaborate description of the system model and the detailed algorithms. The discovery probability analysis for the algorithms and the simulation follows in section III. In section IV, the calculated performance comparison of these algorithms will be illustrated, and we will show the great benefits of the proposed I-SBA. Finally, we conclude the paper in section V.

This work was supported in part by the funds of MIIT of China (2011ZX03001-007-03), the 111 Project (B07022) and the Shanghai Key Laboratory of Digital Media Processing and Transmissions; and it was partially supported by Shanghai Science and technology Development Funds (10QA1403600).

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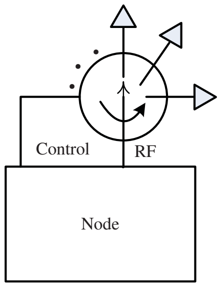


Fig. 1. Node architecture with sectored-antennas.

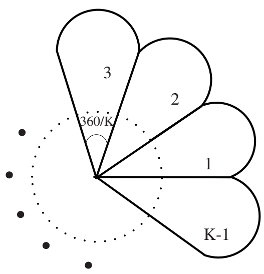
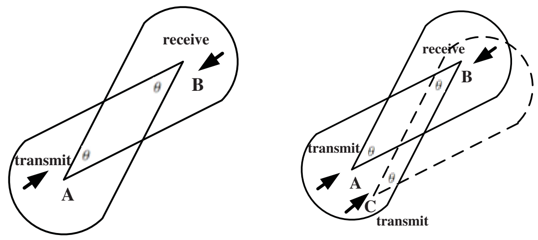


Fig. 2. Idealized K sectored-antennas.



(a) Detection Model (b) Collision Model

Fig. 3. Examples of the nodes communicating successfully and occurring collisions.

II. System Model and Neighbor Discovery Algorithms

A. System Model

In this paper, it is assumed that each node is equipped with a directional antenna with beamwidth θ ($0 < \theta < 2\pi$). All nodes have the fixed transmission power and the same transmission range. Each antenna is a steerable, sectored-antenna where a fixed number of fixed beamwidth antenna elements are mounted to cover the whole azimuth, which is shown in Fig. 1. Sectored-antenna is one of the simplest and the most general realizations of directional antenna. Our conclusion based on this antenna model can be easily extended to the other cases when using different kinds of directional antennas. Switching sectors is done by simply selecting an antenna element as shown in Fig. 2. Moreover, nodes possess the following features:

1) Time is slotted and nodes are perfectly synchronized on time slots by GPS or other methods, every time slot is divided into two minislots.
2) Communication is half-duplex. At any time, a node can either be in transmit or receive state, but not both.
3) A pair of nodes can communicate directly (no relay needed) provided that the straight line connecting them is contained within both the current transmitting beam of one node and the current receiving beam of the other. Fig. 3. (a) shows an example that node A and B communicate successfully.
4) Collisions occur if a node simultaneously receives packets from two or more neighbors. Recovery of the colided packets is impossible. Fig. 3. (b) shows an example that node B receives the signals of node A and node C at the same time, therefore the collision occurs.

B. Neighbor Discovery Algorithms

For 1-way discovery algorithms, a node discovers one neighbor if it receives the discovery message correctly in the receiving state. However, there is no way for the transmitting node to know whether its neighbors receive its message correctly. If omni-directional antennas are used in the system, knowing the location of its neighbors (discovered by receiving only) might be sufficient. However, if directional antennas are used, coordination of antenna steering at both receiver and transmitter ends is required. When a node discovers another node for the first time, they should agree on a future time to communicate again. This process requires feedback from the receivers and results in 2-way (at least) or 3-way handshaking. Here our analysis is based on the 2-way algorithms.

At the start of each time slot, all nodes decide whether to transmit or receive message and in which direction. In every synchronous time slot, if a node is in transmitting mode in the first mini slot, it will transmit an advertisement in a certain direction. In the second mini slot this node will be in listening mode and wait for feedback information in the same direction. On the other hand, with listening mode in the first mini slot, a node waits for advertisements in a certain direction in the first mini slot. If the node receives any advisement information successfully, it will check whether the advertising node has been recorded as its neighbor. In case that the advertising node is a new one, it will respond directionally with its own acknowledgment in the second mini slot. The exchange of messages may also be used to negotiate future rendezvous. Only when a pair of nodes have the successful 2-way handshakes can they successfully discover each other. More details can be found in [16].

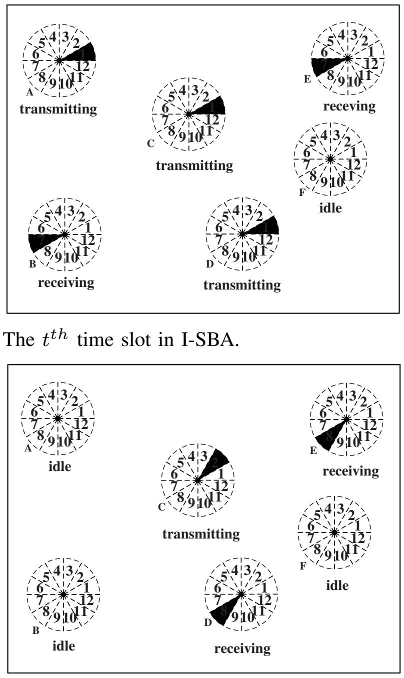
Based on the above description, we define one scan as a sequence (antenna directions) that induces a minimal covering of the entire search volume. For example, in 2-dimension case a scan sequence includes $2\pi/\theta$ beam directions that can cover the entire search volume. The differences between SBA and PRA are actually caused by the different scan sequence they adopt:

1) SBA: each node has the same pre-defined scan sequence of directions for potential neighbors. According to the scan sequence, in each time slot, all nodes transmit messages in the same direction or receive messages in the same opposite direction. We assume that each node transmits with probability p_{t1} or receives with probability $1 - p_{t1}$ in the first mini slot.
2) PRA: the scan sequence of each node is randomly generated. Therefore, at each time slot, nodes would transmit or receive in different directions. We assume that each node transmits with probability p_{t2} or receives with probability $1 - p_{t2}$ in the first mini-slot.

$$Psuc_1(t) = 2p_{t1} \cdot (1 - p_{t1}) \cdot (1 - p_{t1})^{M-1} \cdot p_{t1}^{M-D(t-1)-1} \tag{1}$$

$$Psuc_2(t) = 2p_{t2}\frac{\theta}{2\pi} \cdot (1 - p_{t2})\frac{\theta}{2\pi} \cdot (1 - p_{t2}\frac{\theta}{2\pi})^{M-1} \cdot (1 - (1 - p_{t2})\frac{\theta}{2\pi})^{M-D(t-1)-1} \tag{2}$$

$$Psuc_3(t) = 2p_{t3}^{(1)} \cdot p_{r3} \cdot p_{t3}^{(2)} \cdot (1 - p_{t3}^{(1)})^{M-1} \cdot [p_{r3} \cdot (1 - p_{t3}^{(2)}) + (1 - p_{r3})]^{M-D(t-1)-1} \tag{3}$$



(a) The t^{th} time slot in I-SBA.

(b) The $(t+1)^{th}$ time slot in I-SBA.

Fig. 4. Illustration of I-SBA.

Our proposed I-SBA is an effective improvement of SBA, where one extra mode named 'idle' is added in every time slot. In every first mini slot, nodes can be in transmission, reception or idle mode with probability $p_{t3}^{(1)}$, p_{r3} or $1 - p_{t3}^{(1)} - p_{r3}$ respectively. If a node is in idle mode in the first mini slot, then it will also be idle in the second mini slot. Besides, when the node receives a new node's advisement information successfully in the first mini slot, it will not always respond but with probability $p_{t3}^{(2)}$ in the second mini slot. This scheme will decrease collisions if the probabilities $p_{t3}^{(1)}, p_{r3}$ and $p_{t3}^{(2)}$ are selected properly.

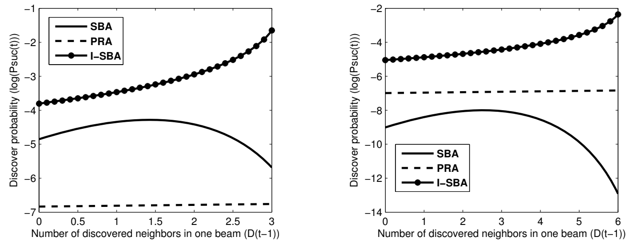
An illustration of the algorithm is shown in Fig. 4. The scan directions of each node includes 12 beam directions, each node has the same scan sequence $\{1, 2, 3...12\}$. At the t^{th} time slot, node A, C and D are transmitting in direction 1. Node E is listening in direction 7, which is opposite to direction 1. Node F is in idle state. At the $(t+1)^{th}$ time slot, node C is transmitting in the direction 2. Node D and E are listening in direction 8, which is opposite to direction 2. Meanwhile Node A, B and F is in idle state. We can see that node C and E can discover each other at the t^{th} time slot.

III. Discovery Probability Analysis and Simulation

A. Discovery Probability Analysis

Assuming that in a wireless network nodes are uniformly distributed and the total number of neighbors of every node is $N = M \cdot \frac{2\pi}{\theta}$, where M is the average number of neighbors in one beam of a node and θ is the directional antenna beamwidth. For each node, the discovery process in every beam direction is independent. The discovery probability in one beam direction of each node is considered and it can conclude the same results in the other beam directions. We define $D(t)$ as the number of neighbors that has been discovered successfully by a node in one beam direction within t time slots ($D(t) \leq M$). In 2-way directional neighbor discovery algorithm, node j discovers node i at the t^{th} time slot if the followings are true (node j is in receiving mode at the beginning of the slot):

1) Node i transmits in the direction of node j in the first mini slot with probability p_{t1} in SBA, $p_{t2} \cdot \frac{\theta}{2\pi}$ in PRA and $p_{t3}^{(1)}$ in I-SBA.
2) Node j receives in the direction of node i in the first mini slot with probability $1 - p_{t1}$ in SBA, $(1 - p_{t2})\frac{\theta}{2\pi}$ in PRA and p_{r3} in I-SBA.
3) Node j transmits in the direction of node i in the second mini slot with probability 1 in SBA and PRA, and $p_{t3}^{(2)}$ in I-SBA. Node i receives directionally in the second mini-slot with probability 1.
4) None of the $M - 1$ neighbors in the node j's beam on i's direction interferes with the transmission from i to j in the first mini slot with probability $(1 - p_{t1})^{M-1}$ in SBA, $(1 - p_{t2}\frac{\theta}{2\pi})^{M-1}$ in PRA and $(1 - p_{t3}^{(1)})^{M-1}$ in I-SBA.
5) None of the $M - 1$ neighbors in the node i's beam on j's direction interferes with the transmission from j to i in the second mini slot. The nodes that have been discovered by node i would not send feedback information to node i if they receive information from node i in the first mini slot. Therefore it only requires that none of the $M - D(t-1) - 1$ neighbors in the node i's beam on j's direction interferes with the transmission from j to i in the second mini slot. In SBA and PRA the event equals that none of the $M - D(t-1) - 1$ neighbors in the node i's beam on j's direction receives in the direction of node i in the first mini slot with probability $p_{t1}^{M-D(t-1)-1}$ in SBA and $(1 - (1 - p_{t2})\frac{\theta}{2\pi})^{M-D(t-1)-1}$ in PRA. In I-SBA the event equals that any one of the



(a) M=4 (b) M=7

Fig. 5. Discover probability for varying number of discovered neighbors ($\theta = 0.1\pi$).

$M - D(t-1) - 1$ neighbors in the node i's beam on j's direction didn't receive in the direction of node i in the first mini slot with probability $1 - p_{r3}$ or received but doesn't send feedback information to node i in the second mini slot with probability $p_{r3}(1 - p_{t3}^{(2)})$.

Combining all the five events and noting that node i and j can discover each other if either node j is in receiving mode or node i is in receiving mode in the first mini slot. Therefore, the discover probability $Psuc_k(t)$ $(k = 1, 2, 3)$ that node i can discover one of its unknown neighbor, node j, in the t^{th} time slot for the three different algorithms, can be presented by (1)-(3), respectively.

Let $Psuc(t)$ denotes $Psuc_k(t)$ $(k = 1, 2, 3)$, the probability that node i discovers node j within t time slots is given by:

$$P_{i,j}(t) = 1 - \prod_{k=1}^{t}(1 - Psuc(k)) \tag{4}$$

The transmission and reception probability can be selected in the beginning of every time slot so that $Psuc(t)$ is maximized. Take derivation of (4) and equate it to 0, the optimal transmission or reception probabilities are as follows:

$$p_{t1} = \frac{M}{2M - D(t-1)}. \tag{5}$$

$$p_{t2} \approx 0.5. \tag{6}$$

$$p_{t3}^{(1)} = \frac{1}{M};$$
$$p_{r3} \cdot p_{t3}^{(2)} = \frac{1}{M - D(t-1)}, \text{When } D(t-1) < M - 1;$$
$$p_{r3} = \frac{M-1}{M}, p_{t3}^{(2)} = 1, \text{When } D(t-1) = M - 1. \tag{7}$$

Fig. 5 plots $Psuc(t)$ as function of $D(t-1)$ with optimal transmission probabilities. It can be seen that I-SBA always has the largest discover probability. More analysis and discussions will be presented in Section IV.

B. Practical Considerations

In Section III-A it is seen that in SBA or I-SBA every node's selection of the optimal transmission or reception probabilities should be based on the prior information of the number of neighbors. In practice, a node may not have exact information about the number of neighbors it has. However transmission or reception probability could be chosen based on some estimation of the expected density of the wireless network $\bar{\gamma}$. This estimation is easily available since $\bar{\gamma}$ can be "wired" into the nodes before deployment. Thus the expected number of neighbors of a node is given by: $\bar{N} = \bar{\gamma}\pi r^2$, where r is the transmission radius of the node. When $\bar{N} > N$, overestimation of the transmission probability will lead to more collisions, while underestimation occurs when $\bar{N} < N$ thereby underutilizing the channel and missing opportunities to discover neighbors.

We define the average discovery probability $\bar{P}suc(t)$ as follows:

$$\bar{P}suc(t) = \sum_{K=0}^{M-1} Psuc(t|D(t-1) = K)/M. \tag{8}$$

In Fig. 6, we plot $\bar{P}suc(t)$ as a function of the estimation error $\bar{N} - N$, when $N = 120$ and $\theta = 0.1\pi$. It is observed that $\bar{P}suc(t)$ is maximized when there is no estimation error and decreases as the error increases either due to underestimation or overestimation. It is seen that in SBA an overestimation of the number of neighbors results in a larger $\bar{P}suc_1(t)$ while in I-SBA an underestimation will bring better results. Similar behaviors are observed for other choices of N and θ. The key observation, however, is that discovery can still be achieved even if there is an error in estimating the number of neighbors and that performance degrades gracefully with increasing error in SBA and decreasing error in I-SBA.

C. Validation of Model

The analysis of the neighbor discovery algorithm was based on the assumption that all nodes belong to a single clique. In reality, network topologies usually are arbitrary and multihop. The model also assumes that the probability of a node i discovering another node j in a time slot is not independent of another node k discovering j in the same time slot. In order to validate these assumptions, we compute the expected number of discovered neighbors within t time slots in one beam, $E[D(t)]$, and compare it with the results obtained by simulation using OPNET. A node i has discovered m ($m = 0, 1.., M$) of its neighbors within t time slots in one beam in the following two ways:

1) Node i has discovered m neighbors in the first $t-1$ time slots and discovers none of the remaining $M - m$ neighbors in the t^{th} time slot.
2) Node i has discovered $m-1$ neighbors in the first $t-1$ time slots and discovers another one of the remaining $M - m - 1$ neighbors in the t^{th} time slot.

Hence, the probability that one node, i, discovers m neighbors within t slots in one beam, denoted by $P(m, t)$ is given by:

$$P(m, t) = k_1 \cdot P(m, t-1) + k_2 \cdot P(m-1, t-1) \tag{9}$$

where

$$k_1 = 1 - (M - m) \cdot Psuc(t|D(t-1) = m),$$
$$k_2 = (M - m + 1) \cdot Psuc(t|D(t-1) = m - 1). \tag{10}$$

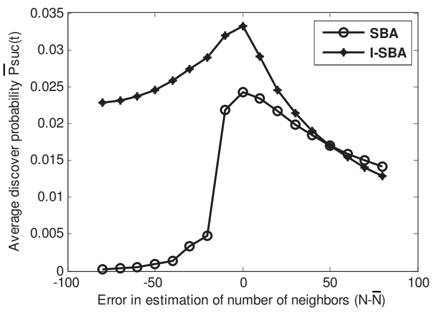


Fig. 6. Effect of estimation error $N - \bar{N}$ on discover probability ($N = 120$ and $\theta = 0.1\pi$).

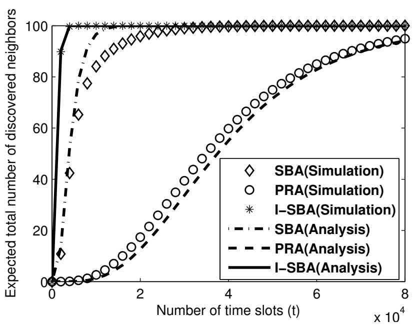


Fig. 7. Validation of analysis ($N = 100$ and $\theta = 0.1\pi$).

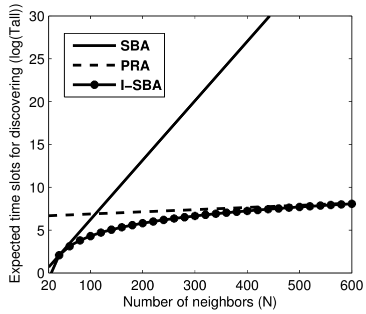


Fig. 8. Expected time slots of discovering all neighbors for varying number of neighbors ($\theta = 0.1\pi$).

Although $P(m, t)$ can not be solved in a closed form, it can be obtained through recurrence since $P(0, 0) = 1$ and $P(1, 0) = 0$. Therefore, the expected number of discovered neighbors within t time slots in one beam, $E[D(t)]$, can be obtained by:

$$E[D(t)] = \sum_{m=1}^{min(M,t)} mP(m, t). \tag{11}$$

Then the expected total number of discovered neighbors within t time slots in all beam directions is $\frac{2\pi}{\theta} E[D(t)]$. We also can solve equation (10) numerically in the same way through recurrence. In order to validate our model, simulation results are obtained in OPNET and compared with the analytical results. In the simulation, 1000 nodes are uniformly located in a square with area $3.14\times10^6 m^2$. Each node has a transmission range $r = 100m$ and a beamwidth $\theta = 0.1\pi$. Therefore the node density $\gamma = \frac{1000}{3.14\times10^6}$ nodes/ m^2. So each node thus has on average $N = \gamma\pi r^2 = 100$ neighbors and $M = 5$ neighbors in every beam direction. The transmission or reception probabilities are obtained from equation (5)-(7) and the other physical parameters are same as specified in the IEEE 802.11b. The comparison between analytical and simulation results is shown in Fig. 7. It can be seen that the numeral results are quite close to the simulation results, which validates our analytical model.

IV. Performance Analysis and Discussions

In this section, we will compare and discuss the performance of these three algorithms by considering the total amount of time slots they take to discover all the neighbors. The key idea is to analyze the discovery process in one beam direction of each node firstly and then conclude the total neighbor discovery latency.

The expectation of time slots required to discover all the neighbors is given by (derivation can be found in Appendix A):

$$T_{all} = \frac{2\pi}{\theta} \sum_{k=0}^{M-1} \frac{1}{(M-k)Psuc(t|D(t-1)=k)}. \tag{12}$$

Similarly, T_{all} can be solved as function of N through recurrence, which is shown in Fig. 8 (beam width $\theta = 0.1\pi$). It can be seen that the performance of SBA is good when the number of neighbors is small, but it degrades quickly when neighbors increases and thus introduces more collisions. On the contrary, PRA is prone to work well when the number of neighbors is large. However, I-SBA has very good performance no matter how many neighbors there are.

The successful neighbor discovery process using directional antennas requires two conditions satisfied, i.e., detection and no collision. SBA makes all nodes transmit in the same directions and receive in opposite directions to increase the detection probability. However, it will exert negative effects when the number of neighbors is large because of introducing more collisions. On the contrary, PRA makes all nodes transmit and receive in different directions that decreases the collision probability but makes detection of nodes more difficult. Therefore, in I-SBA a new state 'idle' is added so that it can equivalently decrease the number of neighbors and effectively diminish the negative influence on collisions. With proper selection of parameters, I-SBA can also remain the superiority of high detection probability of SBA.

V. Conclusions

In this paper, we considered the problem of neighbor discovery in wireless networks with directional antennas, taking into account the collision effects. A new discovery algorithm I-SBA was proposed and a mathematical model was presented to analyze 2-way synchronous directional neighbors discovery algorithms in ad hoc networks. The transmission and reception probabilities were derived to maximize the probability of discovering their neighbors. Simulation of the algorithms demonstrated the validation of our analytical model. Finally, performances of the new I-SBA and the previous algorithms SBA and PRA were compared via numerical analysis. It showed that the new algorithm has enormous benefits over previous ones.

Appendix A

We assume that at t_0^{th} time slot the number of the discovered neighbors in one beam of one node i is $D(t_0) = l$. Let $N_l(t)$ be a random variable that denotes the number of neighbors that can be discovered successfully in this beam by node i from t_0^{th} to $(t_0+t)^{th}$ time slot remaining $D(t_0) = l$. Using the Poisson approximation (assuming large t and small $Psuc_l(t_0|D(t_0))$), it is easy to see that $N_l(t)$ with probability:

$$P(N_l(t) = k) = \frac{e^{-\lambda_l t}(\lambda_l t)^k}{k!} \tag{13}$$

where

$$\lambda_l = (M - l)Psuc(t|D(t_0) = l). \tag{14}$$

Let τ_{l_1} denote the waiting time for the first successful discovery remaining $D(t_0) = l$, which is also the required time slots to discover the next neighbor in this beam when $D(t_0) = l$. For event $\tau_{l_1} < t$ equals event $N_l(t) \geq 1$, the variable τ_{l_1} has a distribution function:

$$F_{\tau_{l_1}}(t) = P(\tau_{l_1} \leq t) = P(N_l(t) \geq 1) = 1 - e^{-\lambda_l t}, t \geq 0. \tag{15}$$

Therefore the p.d.f. of τ_{l_1} can be obtained:

$$f_{\tau_{l_1}}(t) = \begin{cases} \lambda_l e^{-\lambda_l t} & t \geq 0, \\ 0 & t < 0. \end{cases} \tag{16}$$

Assuming that the random variable t_{all} denotes the time slots for discovering all neighbors in this beam, which is given by:

$$t_{all} = \sum_{l=0}^{M-1} \tau_{l_1}. \tag{17}$$

Lemma: the p.d.f. of t_{all} can be obtained through mathematical induction and the result is given by:

$$f_{t_{all}}(u) = \sum_{p=0}^{M-1} \frac{\prod_{q=0}^{M-1} \lambda_q \cdot e^{-\lambda_p u}}{\prod_{q=0,q\neq p}^{M-1}(\lambda_q - \lambda_p)}, u \geq 0. \tag{18}$$

Proof: let $t_k = \sum_{l=0}^{k-1} \tau_{l_1}(k < M)$, then when $k = 1$, $f_{t_1}(t_1) = f_{t_1}(\tau_{0_1}) = \lambda_0 e^{-\lambda_0 t_1}$, $t_1 \geq 0$.

Assuming that when $k = k'(k' \geq 1)$, the following equation holds:

$$f_{t_{k'}}(t_{k'}) = \sum_{p=0}^{k'-1} \frac{\prod_{q=0}^{k'-1} \lambda_q \cdot e^{-\lambda_p t_{k'}}}{\prod_{q=0,q\neq p}^{k'-1}(\lambda_q - \lambda_p)}, t_{k'} \geq 0.$$

Then when $k = k' + 1$,

$$\begin{aligned} & f_{t_{k'+1}}(t_{k'+1}) \\ = \; & f_{t_{k'+1}}(t_{k'} + \tau_{k'_1}) \\ = \; & \int_0^{t_{k'+1}} f_{t_{k'}}(t_{k'}) \cdot f_{\tau_{k'_1}}(t_{k'+1} - t_{k'}) dt'_k \\ = \; & \int_0^{t_{k'+1}} \sum_{p=0}^{k'-1} \frac{\prod_{q=0}^{k'-1} \lambda_q \cdot e^{-\lambda_p t_{k'}}}{\prod_{q=0,q\neq p}^{k'-1}(\lambda_q - \lambda_p)} \lambda_{k'} e^{-\lambda_{k'}(t_{k'+1} - t_{k'})} dt_{k'} \\ = \; & \int_0^{t_{k'+1}} \sum_{p=0}^{k'-1} \frac{\prod_{q=0}^{k'} \lambda_q}{\prod_{q=0,q\neq p}^{k'-1}(\lambda_q - \lambda_p)} e^{-\lambda_{k'} t_{k'+1}} e^{t_{k'}(\lambda_{k'} - \lambda_p)} dt_{k'} \\ = \; & \sum_{p=0}^{k'} \frac{\prod_{q=0}^{k'} \lambda_q \cdot e^{-\lambda_p t_{k'+1}}}{\prod_{q=0,q\neq p}^{k'}(\lambda_q - \lambda_p)} \end{aligned} \tag{19}$$

From equation (19) we can get the lemma proved.■

Therefore, the expectation of time slots required to discover all the neighbors is given by:

$$\begin{aligned} E[T_{all}] &= \frac{2\pi}{\theta} E[t_{all}] \\ &= \frac{2\pi}{\theta} \int_0^{+\infty} \tau f(t_{all} = \tau) d\tau \\ &= \frac{2\pi}{\theta} \sum_{l=0}^{M-1} \frac{1}{\lambda_l}. \end{aligned} \tag{20}$$

Combining equation (14) and (20), we have equation (12).

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