Minimum Length Scheduling in Single-hop Multiple Access Wireless Networks

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***Abstract*—In this paper, we address the minimum length scheduling problem in wireless networks, where each transmitter has a finite amount of data to deliver to a common receiver node (e.g., base station). In contrast with previous works that model wireless channels according to the *Protocol* or *Physical* model of interference, this paper studies the scheduling problem in multiple access (multi-access) networks. In this kind of network, the receiver node can decode multiple transmissions simultaneously if the transmission rates of concurrent transmitters lie inside the capacity region of the receiver node. We propose a linear programming model that minimizes the schedule length. The model incorporates the capacity region of multiple access channels into scheduling decisions, such that the sum of the transmission rates of simultaneous transmitters is maximized. Because of the high-complexity of the model, we also present a heuristic algorithm, whose performance is shown to be *close* to optimal according to our simulation results.**

I. Introduction

Recent advances in signal detection techniques open up new opportunities for resolving collisions at the physical layer. These techniques permit the simultaneous reception of multiple transmissions by a receiver node, which increases the capacity of wireless networks [1]. In this kind of network, called multiple access (multi-access) network, the signals from multiple transmitters are decoded by exploiting multiuser techniques such as Successive Interference Cancelation (SIC) and Code Division Multiple Access (CDMA). The multi-access capability at physical layer calls for the design of new Medium Access Control (MAC) schemes tailored for this capability.

Conflict-free scheduling, as an access control, avoids collisions and retransmissions that are typical in contention-based methods [2]. Whereas typical scheduling schemes such as Time Division Multiple Access (TDMA) are simple and easy to implement, they do not exploit multi-access capability, which leads to suboptimal channel usage. In order to improve throughput, the design of multi-access schemes has received tremendous attention over the last years. Recent work [1], [3] showed that the combination of multi-access techniques at the physical layer and conflict-free schemes at the MAC layer has the potential to significantly improve the performance of the network. Subsequent work considered alternative simple schemes to exploit multiple access capability, where nodes transmit at a homogeneous transmission rate [4], [5]. Further work dealing with contention-free scheduling includes [2], [4], [6], [7]. Celik et al. [4] studied the negative implications of reusing legacy MAC protocols in multi-access networks, and how alternative backoff mechanisms can improve throughput and fairness. Borbach and Ephremides [8] proposed a linear program to minimize the schedule length under the Physical model of interference, where transmission rates are fixed and the power is optimized. In [2] and [6], Kompella et al. also proposed a linear program under the Physical model of interference for the minimum length schedule, which has a prohibitively large number of variables. Additionally, the authors presented a column generation approach. Pantelidou and Ephremides [7] solved the minimum length schedule problem under the Physical model of interference by reducing it to the shortest path problem in a graph. The authors characterized the optimal scheduling when transmission rates and scheduling orders are restricted to a subset of the total possible options.

In this paper, we present a linear program for the *Minimum Length Scheduling Problem* (MLSP) in single-hop multiple access wireless networks. Given a set of transmitter nodes and corresponding traffic demands, MLSP minimizes the completion time or schedule length required to satisfy the specified traffic demands. By minimizing the amount of time to send a fixed amount of data, the throughput is effectively maximized. The model incorporates the capacity region of multi-access channels into scheduling decisions, such that the sum of the transmission rates of simultaneous transmitters is maximized. Because of the high-complexity of the model, we also present a heuristic algorithm, whose performance is shown to be *close* to optimal according to our simulation results.

The remainder of the paper is organized as follows. Section II overviews the multiple access channel model and its capacity region. Section III formulates MLSP as a linear program, and Section IV presents a heuristic for MLSP. Section V shows performance studies, and Section VI concludes our work.

II. Multiple Access Channel

We consider a set $\mathcal{T} = \{1, 2, ..., n\}$ of transmitter nodes ready to transmit to a common receiver node, where $|\mathcal{T}| = n$ and $|\cdot|$ denotes cardinality. Transmitter nodes transmit at a common power level P_0. Let d_j be the distance between a transmitter node $j \in \mathcal{T}$ and the receiver node. The corresponding received power P_j at the receiver location decays exponentially with d_j:

$$P_j = P_0 d_j^{-\gamma}, \quad (1)$$

where γ is the path loss exponent.

We assume an Additive White Gaussian Noise (AWGN) channel, where the variance of the channel noise is denoted as η. Define the channel capacity function of a single user of an AWGN channel with bandwidth W and signal to interference plus noise ratio $SINR$ as:

$$\varphi(SINR) = W \log_2(1 + SINR). \quad (2)$$

Let $S \subseteq \mathcal{T}$ be a *schedulable set*, a set of nodes scheduled to simultaneously transmit at a time. Let $r_j(S)$ be the transmission rate in bits per second (bps) of node $j \in S$ when simultaneously scheduled with all transmitters in S, and $\vec{r}(S)$ be the rate vector for such set. To successfully decode all transmissions scheduled in S, the rates at which transmitter nodes send data must lie inside the capacity region of the multiple access channel. The region is defined as the closure of the convex hull of rate vectors $\vec{r}(S)$ satisfying:

$$\sum_{j \in S'} r_j(S') \quad \leq \quad \varphi\left(\frac{\sum_{j \in S'} P_j}{\eta}\right), \quad (3)$$

for all $S' \subseteq S$ [9]. The region given by Eq. (3) is characterized by $2^{|S|} - 1$ constraints, each corresponding to a nonempty subset of transmitters. The capacity region has precisely $|S|!$ vertices in the positive quadrant, each achievable by SIC using one of the $|S|!$ possible orderings. The following example illustrates the case for $|S| = 2$.

Example 1. Assume that two nodes 1 and 2 are scheduled to simultaneously transmit to a node 0; i.e., $S = \{1, 2\}$. The capacity region of node 0, which is shown in Fig. 1, is reduced to the following three constraints:

$$r_1(S) \leq \varphi\left(\frac{P_1}{\eta}\right), r_2(S) \leq \varphi\left(\frac{P_2}{\eta}\right), r_1(S) + r_2(S) \leq \varphi\left(\frac{P_1 + P_2}{\eta}\right).$$

The vertices of the capacity region are labeled as $\vec{r_a}, \vec{r_b}, \vec{r_c}$, and $\vec{r_d}$. The aggregate rate, $r_1(S)+r_2(S)$, is maximized when a rate vector lies in the segment line between $\vec{r_b}$ and $\vec{r_c}$. The points $\vec{r_b}$ and $\vec{r_c}$ can be achieved by using SIC and CDMA. For example, $\vec{r_c}$ can be obtained in a two-stage SIC decoding process. In the first stage, node 0 decodes packet p_1 from node 1, considering the transmission from node 2 as part of noise. Therefore, $r_1(S)$ can be as high as $\varphi\left(\frac{P_1}{\eta+P_2}\right)$. In the second stage, after packet p_1 has been decoded, it can be subtracted out; therefore, if packet p_2 from node 2 is sent at a rate $\varphi\left(\frac{P_2}{\eta}\right)$, it can be successfully decoded. The packets are sent using different codes (CDMA). In general, the capacity region achieved with CDMA and SIC is larger than that achieved with TDMA, which is bounded by the dashed line in Fig. 1 [9].

From *Example 1*, it is important to highlight that the order in which simultaneous transmissions are decoded may impact on the length of a schedule. In the previous example, the transmitters constitute the schedulable set $S_1 = \{1, 2\}$ (transmission from node 1 is decoded first), and operate at rate vector $\vec{r_c}$. On the other hand, for a schedulable set $S_2 = \{2, 1\}$ (transmission from node 2 is decoded first), the same transmitter nodes operate at a different point, namely, rate vector $\vec{r_b}$. In general, even though the sum of the rates at the vertices of the positive quadrant is the same, the permutations of transmitter nodes represent different decoding orders, which may produce different schedule lengths. Without loss of generality, when representing a schedulable set $S = \{v_1, v_2, ..., v_{|S|}\}$, $v_i \in \mathcal{T}$, we will assume that the decoding process follows this order.

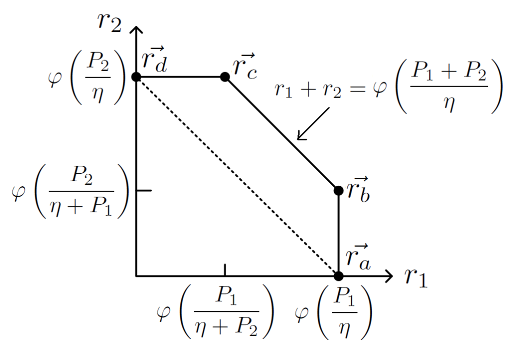

Fig. 1. Capacity region of a multi-access channel for a receiver node receiving data from two simultaneous transmitters. The region is upper bounded by solid lines. For a single user channel model, the capacity region is upper bounded by the dashed line.

For a general number of transmitters, the points in the capacity region that maximize the sum of the simultaneous transmission rates are defined as follows.

Definition 1: Let S be a set of nodes simultaneously transmitting to a common receiver node. Transmitter nodes are said to operate at sum-rate if:

$$\sum_{j \in S} r_j(S) = \varphi\left(\frac{\sum_{j \in S} P_j}{\eta}\right).$$

Since the maximum transfer of information is achieved at sum-rate, clearly any scheduling strategy should schedule transmissions such that transmitter nodes operate at sum-rate. As we shall see, the length of a schedule is minimized when concurrent transmitters operate at these points. At the same time, increasing the number of simultaneous transmitters would minimize the schedule length. However, in practice, a receiver node can decode only a certain number K of simultaneous transmissions [1], [4]. We will refer to this limitation K as the *decoding capability* of a receiver node. Complexity and energy consumed by decoders are main limitations that restrict the decoding capability. Thus, MLSP in multiple access networks deals with scheduling transmitters in groups of no more than K nodes, such that each transmitter satisfies its traffic demand in a minimum period of time.

III. Problem Formulation

A. Linear Program

Assume that simultaneous transmitters operate at sum-rate. Each transmitter $j \in \mathcal{T}$ has a specific traffic demand of f_j bits to be transmitted to the receiver node. The set of all traffic demands is denoted as $F = \{f_1, f_2, ..., f_n\}$, and the decoding capability of the receiver node is denoted as K. Thus, at any time, the number of active transmitters is equal to K or less. Let $\Gamma = \{S_1, S_2, ..., S_{|\Gamma|}\}$ be the set of all schedulable sets operating at sum-rate, $S_i \subseteq \mathcal{T}$ for all i; note that the size of any schedulable set in Γ is at most K. Additionally, as stated in Section II, schedulable sets are permutations of transmitter nodes. Let $r_j(S)$ be the transmission rate at which transmitter node $j \in S$ operates when scheduled in schedulable set $S \in \Gamma$. Let $\tau(S)$ be a variable representing the amount of time that schedulable set S is activated. The linear program for the minimum length scheduling problem (LP-MLSP) in single-hop multi-access networks is defined in Fig. 2.

$$\min \quad T \tag{4}$$

$$\sum_{S\in\Gamma: j\in S} \tau(S) r_j(S) \geq f_j; \quad j \in \mathcal{T} \tag{5}$$

$$\sum_{S\in\Gamma} \tau(S) = T \tag{6}$$

$$\tau(S) \geq 0; \quad S \in \Gamma \tag{7}$$

Fig. 2. Linear program for the MLSP in multi-access networks (LP-MLSP).

Eq. (4) is the schedule length, which must be minimized. Eq. (5) represents the traffic demand constraint; for each transmitter node $j \in \mathcal{T}$, the amount of data that node j can send to the receiver node must be greater or equal to its traffic demand f_j. Eq. (6) states that the schedule length is equal to the total amount of time allocated to all schedulable sets. Eq. (7) restricts the time allocated to each schedulable set to be non-negative.

B. Complexity of LP-MLSP

To characterize LP-MLSP, we will embed it in a vector space $\{0,1\}^n$. Let $\vec{S}$ be the *characteristic vector* of the schedulable set $S \in \Gamma$. The j^{th} element of this vector is set to one if transmitter node $j \in \mathcal{T}$ is a member of S, and to zero otherwise. Any characteristic vector $\vec{S}$ can be regarded as a point in an n-dimensional space, which also becomes a vertex of the the convex hull of the set of characteristic vectors. Let $\lambda(S)$ be the *normalized fraction of time* that schedulable set $S \in \Gamma$ is activated:

$$\lambda(S) = \frac{\tau(S)}{T}. \tag{8}$$

Let $\vec{u} = (u_1, u_2, ..., u_n)$ be an n-dimensional *utilization vector*, where u_j indicates the total fraction of time that transmitter node j is activated. By regarding $\vec{u}$ as a point in $\{0,1\}^n$, then we have the following proposition.

Proposition 1: Let $Co(\Gamma)$ be the convex hull of all possible characteristic vectors. A solution of LP-MLSP given by a set $\Gamma' = \{S_1, S_2, ..., S_{|\Gamma'|}\} \subseteq \Gamma$ with corresponding normalized allocation times $\lambda(S_1), \lambda(S_2), ..., \lambda(S_{|\Gamma'|})$ is feasible iff the resulting utilization vector $\vec{u}$ lies within $Co(\Gamma)$.

For the proof of *Proposition 1*, please refer to [10]. The following example illustrates *Proposition 1*.

Example 2. Consider Fig. 3(a), where $\mathcal{T} = \{1,2,3\}$, and the traffic demands are f_1, f_2, and f_3. Assume first that $K = 1$ (i.e., a TDMA system). The only schedule that activates the three nodes includes $S_1 = \{1\}, S_2 = \{2\}$, and $S_3 = \{3\}(\Gamma' = \{S_1, S_2, S_3\})$. The constraints given by Eq. (5) are:

$$\tau(S_1)\varphi\left(\frac{P_1}{\eta}\right) \geq f_1, \tau(S_2)\varphi\left(\frac{P_2}{\eta}\right) \geq f_2, \tau(S_3)\varphi\left(\frac{P_3}{\eta}\right) \geq f_3.$$

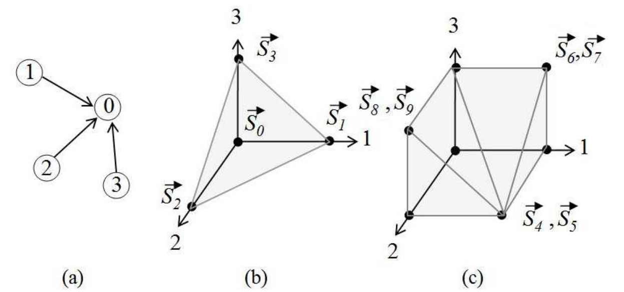

Fig. 3. (a) Network topology with three transmitter nodes and a receiver node. (b) Convex hull of characteristic vectors, assuming $K = 1$. (c) Convex hull of characteristic vectors, assuming $K = 2$.

The schedule length given by Eq. (6) is expressed as: $\tau(S_1) + \tau(S_2) + \tau(S_3) = T$. Equivalently, Eq. (8) and *Proposition 1* require that the resulting utilization vector lies inside the polytope shown in Fig. 3(b). Assume now a multi-access channel with $K = 2$. Any permutation of size 2 is schedulable; i.e., $S_4 = \{1,2\}$, $S_5 = \{2,1\}$, $S_6 = \{1,3\}$, $S_7 = \{3,1\}$, $S_8 = \{2,3\}$, and $S_9 = \{3,2\}$ are feasible. Note that the characteristic vectors are not unique (except for permutations of size 1); e.g., $\vec{S_4} = \vec{S_5} = (1,1,0)$. Let $\Gamma' = \{S_3, S_5\}$ be a feasible solution, where for S_3, transmitter node 3 operates at sum-rate $r_3(S_3) = \varphi\left(\frac{P_3}{\eta}\right)$. Similarly, for S_5, transmitter nodes 2 and 1 (whose transmissions are decoded in this order) operate at sum-rate, $r_2(S_5) = \varphi\left(\frac{P_2}{\eta+P_1}\right)$ and $r_1(S_5) = \varphi\left(\frac{P_1}{\eta}\right)$ (see Fig. 1, rate vector $\vec{r_b}$). The constraints given by Eq. (5) are:

$$\tau(S_5)\varphi\left(\frac{P_1}{\eta}\right) \geq f_1, \tau(S_5)\varphi\left(\frac{P_2}{\eta+P_1}\right) \geq f_2, \tau(S_3)\varphi\left(\frac{P_3}{\eta}\right) \geq f_3.$$

The polytope for $K = 2$ is shown in Fig. 3(c).

Even though LP-MLSP is a linear program with a simple constraint structure, *Example 2* shows that its complexity increases as the decoding capability increases. LP-MLSP can be seen as mixed continuous-discrete optimization problem. The optimization variables $\tau(S)$'s are continuous variables representing the amount of time allocated to the schedulable sets. On the other hand, the extreme points of the constraint polytope are integers and represent solutions of a combinatorial problem, namely, feasible schedulable sets. The number of extreme points, $\sum_{i=0}^{K} \binom{n}{i}$, represents the number of combinations of transmitter nodes of size at most K. Moreover, since each extreme point is mapped to more than one schedulable set for $K > 1$ (because every different decoding order may produce a different schedule length), LP-MSLP must at least consider all different permutations of size K (smaller permutations can be neglected; the sum-rate of a large permutation will always be greater than that of a smaller permutation. The size of the problem, therefore, can be accordingly reduced). Let P_K be the number of permutations of size K. This number can be lower bounded as follows:

$$P_K = n(n-1)(n-2)...(n-K+1) = \binom{n}{K} K!$$

Heuristic Scheduler (HS)

1: INPUT: $\mathcal{T}$, F, node locations;
2: OUTPUT: Set $\Gamma_{HS} = \{S_1, S_2, ..., S_{|\Gamma_{HS}|}\}$ of schedulable sets with associated transmission rates $r_{v_i}(S)$, $\forall v_i \in S, \forall S \in \Gamma_{HS}$;

3: $f'_j = f_j, \forall j \in \mathcal{T}$;
4: $\Gamma_{HS} = \{\}$; $S_0 = \{\}$; $t = 0$; $i = 0$;
5: **while** $(\exists\, j | f'_j > 0)$ **do**
6:     **while** $(\exists\, j \in \mathcal{T} | S_i \cup \{j\}$ is maximal$)$ **do**
7:         $S_i = S_i \cup \{j\}$;
8:         $\mathcal{T} = \mathcal{T} \backslash j$;
9:     **end while**
10:     Select transmission rates according to Eq. (11);
11:     $\Gamma_{HS} = \Gamma_{HS} \cup S_i$;
12:     $v = \text{argmin}\left\{\frac{f'_v}{r_v(S_i)}\right\}$;
13:     $\tau(S_i) = \frac{f'_v}{r_v(S_i)}$;
14:     $f'_j = f'_j - \tau(S_i), \forall j \in S_i$;
15:     $i = i + 1$;
16:     $S_i = S_{i-1} \backslash v$;
17: **end while**

Fig. 4. Heuristic Scheduler (HS).

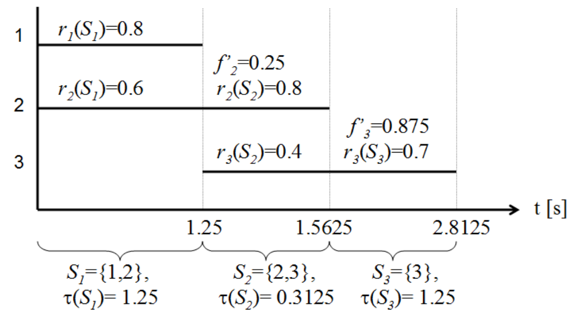

Fig. 5. Operation of HS, for $\mathcal{T} = \{1, 2, 3\}$, $f_1 = f_2 = f_3 = 1$ Mbits, and $K = 2$. HS builds $S_1 = \{1, 2\}$; assuming operation rates $r_1(S_1) = 0.8$ Mbps and $r_2(S_1) = 0.6$ Mbps (rates are selected according to Eq. (11)), node 1 satisfies its demand at $t = 1.25$ s. HS then removes node 1 from S_1 and schedules node 3, so that $S_2 = \{2, 3\}$. The residual traffic demand $f'_2 = 0.25$ Mbits. Assuming that $r_2(S_2) = 0.8$ Mbps and $r_3(S_2) = 0.4$ Mbps, node 2 satisfies its residual demand at $t = 1.5625$ s. HS proceeds to remove node 2 from S_2 and schedules $S_3 = \{3\}$ until node 3 satisfies its residual demand at $t = 2.8125$ s., which represents the schedule length.

$$
\begin{aligned}
&= \frac{n(n-1)...(n-K+1)}{K(K-1)...1}K! = \frac{n}{K}\frac{n-1}{K-1}...\frac{n-K+1}{1}K! \\
&\geq \left(\frac{n}{K}\right)^K K! >^{(a)} \left(\frac{n}{K}\right)^K 2^K = c_1^K, \qquad (9)
\end{aligned}
$$

where (a) is true for $K > 3$, and $c_1 = \frac{2n}{K} > 1$. Similarly, an upper bound for P_K is:

$$P_K =^{(a)} \frac{n(n-1)...(n-K+1)}{K(K-1)...1}K! \leq \frac{n^K}{K!}K! = n^K, \qquad (10)$$

where (a) follows from the second line of the derivation of Eq. (9). Since each variable in Eq. (7) corresponds to a schedulable set (i.e., a permutation of size K of transmitter nodes), the total number of variables of LP-MLSP is upper and lower bounded by Eqs. (9) and (10). These bounds are not polynomial in the number of transmitter nodes, and thus cannot be efficiently solved for large values of K and n. Additionally, it is not hard to see that the minimum makespan scheduling problem [11] (which is NP-hard) can be reduced to LP-MLSP. Thus, as an alternative solution, in next section we propose a heuristic algorithm for LP-MLSP.

IV. Heuristic Algorithm for LP-MLSP

A. Algorithm Description

We present a low-complexity polynomial time algorithm for LP-MLSP. The algorithm is shown in Fig. 4 and is denoted as Heuristic Scheduler (HS). HS proceeds as follows: it schedules transmitter nodes in an arbitrary order, one by one, until the number of scheduled transmitters is equal to K. The obtained set S_1 represents a maximal[1] schedulable set. Let $v_1, v_2, ..., v_K$ be the order in which transmitter nodes are scheduled. HS selects the corresponding transmission rates as:

$$
\begin{aligned}
r_{v_1}(S_1) &= \varphi\left(\frac{P_{v_1}}{\eta}\right), r_{v_2}(S_1) = \varphi\left(\frac{P_{v_2}}{\eta + P_{v_1}}\right), ..., \\
r_{v_K}(S_1) &= \varphi\left(\frac{P_{v_K}}{\eta + \sum_{i=1}^{K-1} P_{v_i}}\right). \qquad (11)
\end{aligned}
$$

[1]A set that is maximal under inclusion is called a maximal set.

Let $v = \text{argmin}\left\{\frac{f_{v_i}}{r_{v_i}(S_1)}\right\}$; v represents the transmitter node that first satisfies its traffic demand at transmission rate $r_v(S_1)$ (Fig. 4, line 12). HS allocates to S_1 an amount of time $\tau(S_1) = \frac{f_{v_i}}{r_{v_i}(S_1)}$. After S_1 is scheduled, the residual traffic demand is given by $f'_{v_i} = f_{v_i} - \tau(S_1) r_{v_i}(S_1), \forall v_i \in S_1$. HS proceeds by creating a new schedulable set $S_2 = S_1 \backslash v \cup \{u\}$, where $u \in \mathcal{T}$ is a transmitter node not previously scheduled. The transmission rates for nodes in S_2 are also computed with Eq. (11); they are selected according to the order in which nodes were scheduled. Consider a general iteration, when S_i is being created, and sets $S_1, ..., S_{i-1}$ have already been built. The residual traffic demand of a node v is:

$$f'_v = f_v - \sum_{\forall j \in \{1,2,...,i-1\}: v \in S_j} r_v(S_j)\tau(S_j). \qquad (12)$$

The time allocated to S_i can be written as:

$$\tau(S_i) = \min\left\{\frac{f'_v}{r_v(S_i)} : v \in S_i\right\}. \qquad (13)$$

The process is repeated until all nodes have satisfied their traffic demands. Fig. 5 illustrates the operation of HS.

B. Transmission Rates and Complexity

The following proposition states that nodes operate at sum-rate, and therefore the corresponding rates are achievable.

Proposition 2: Let $S = \{v_1, v_2, ..., v_H\}$ be a set of transmitter nodes scheduled by HS in this order, where $H \leq K$. For the transmission rates given by Eq. (11), the corresponding rate vector represents a vertex in the positive quadrant of the capacity region, where the sum of the rates equals sum-rate.

The proof of *Proposition 2* is by induction. Since all transmitters belong to the same schedulable set, we will omit the identifier of the set.
Basis step: assume that transmitters v_1, v_2 are scheduled in this order. Their rates given by Eq. (11) are $r_{v_1} = \varphi\left(\frac{P_{v_1}}{\eta}\right)$

and $r_{v_2} = \varphi\left(\frac{P_{v_2}}{\eta+P_{v_1}}\right)$. By adding the rates we obtain:

$$\begin{aligned} r_{v_1} + r_{v_2} &= W\log_2\left(1+\frac{P_{v_1}}{\eta}\right) + W\log_2\left(1+\frac{P_{v_2}}{\eta+P_{v_1}}\right) \\ &= W\log_2\left(\frac{\eta+P_{v_1}}{\eta}\cdot\frac{\eta+P_{v_1}+P_{v_2}}{\eta+P_{v_1}}\right) \\ &= W\log_2\left(1+\frac{P_{v_1}+P_{v_2}}{\eta}\right), \end{aligned}$$

which proves that nodes v_1 and v_2 operate at sum-rate.

Extension step: assume that nodes $v_1, v_2, ..., v_{H-1}$ were already scheduled by HS and operate at sum-rate:

$$\sum_{h=1}^{H-1} r_{v_h} = W\log_2\left(1+\frac{\sum_{h=1}^{H-1} P_{v_h}}{\eta}\right).$$

HS then schedules the H^{th} transmitter node, which operates at a rate $r_{v_H} = \varphi\left(\frac{P_{v_H}}{\eta+\sum_{h=1}^{H-1} P_{v_h}}\right) = W\log_2\left(1+\frac{P_{v_H}}{\eta+\sum_{h=1}^{H-1} P_{v_h}}\right)$. The sum of the rates of the transmitter nodes is:

$$\begin{aligned} \sum_{h=1}^{H} r_{v_h} &= r_{v_H} + \sum_{h=1}^{H-1} r_{v_h} = W\log_2\left(1+\frac{P_{v_H}}{\eta+\sum_{h=1}^{H-1} P_{v_h}}\right) + \\ & \quad W\log_2\left(1+\frac{\sum_{h=1}^{H-1} P_{v_h}}{\eta}\right) \\ &= W\log_2\left(\frac{\eta+\sum_{h=1}^{H-1} P_{v_h} + P_{v_H}}{\eta+\sum_{h=1}^{H-1} P_{v_h}}\cdot\frac{\eta+\sum_{h=1}^{H-1} P_{v_h}}{\eta}\right) \\ &= W\log_2\left(1+\frac{\sum_{h=1}^{H} P_{v_h}}{\eta}\right), \end{aligned}$$

which demonstrates *Proposition 2*.

The running time of HS can be upper bounded as follows. The outer loop between lines 5 and 17 is executed n times, (one traffic demand is satisfied in each iteration). Similarly, the inner loop between lines 6 and 9 is executed n times (over all outer iterations), since each transmitter node is scheduled once. For each schedulable set S_i, the transmission rates of the nodes belonging to S_i are selected in line 10 according to Eq. (11). Since in each set S_i there are at most $K \leq n$ nodes, evaluating Eq. (11) takes $O(n)$ time. Finally, since line 10 is inside the outer loop, it is executed $O(n)$ times, which gives an upper bound of the running time of $O(n^2)$.

V. Performance Studies

We present numerical examples of MLSP and solutions obtained with both LP-MLSP and HS. LP-MLSP and HS were implemented as solvers in C language. To solve linear programs, the package LP-solve [12] was included. We set the channel bandwidth $W = 1$ MHz, transmission power $P_0 = 1$ W, and path loss exponent $\gamma = 3$. Ten transmitters located in a disk of radio 100 meters (m), centered at the receiver node, were generated. Their traffic demands were uniformly assigned between 1 Mbit and 10 Mbits. We obtained results for scenarios with different channel noise values. As reference, we used the $SNR_{ref} = \frac{P_0 d_{ref}^{-\gamma}}{\eta}$ to setup the value η, where $d_{ref} = 100$ m. For simplicity, from here on, SNR_{ref} will be referred as SNR.

Fig. 6(a) shows the results obtained with two different noise levels: i) with $SNR = -10$ dB (*high-noise* scenario), and ii) with $SNR = 10$ dB (*low-noise* scenario). The curves labeled as LP-MLSP represent optimal performances, and the curves labeled as HS represent the results obtained with the heuristic algorithm. To appreciate the gain with respect to TDMA systems, the results are normalized to that obtained with $K = 1$, where LP-MLSP and HS produce the same results ($K = 1$ implies that transmitter nodes are scheduled in a TDMA or time-sharing manner). Consider first the curves for the scenario with $SNR = -10$ dB. For $K > 2$, the schedule length of LP-MLSP is less than 0.5, which implies a potential throughput improvement of 100% with respect to TDMA. Similarly, the results of HS for $K \geq 4$ are less than 0.5. Consider now the low-noise scenario where $SNR = 10$ dB. Note that the reduction in schedule length, from $K = 1$ (TDMA) to any $K > 1$, is not as significant as that obtained in the high-noise scenario; e.g., for $K = 5$ and $SNR = -10$ dB, the schedule length is reduced up to 0.33 (optimal schedule, given by LP-MLSP); on the other hand, for $K = 5$ and $SNR = 10$ dB, the schedule length is only reduced up to 0.6. This fact can be better appreciated in Fig. 6(b), where the maximum reduction, with respect to TDMA scheduling, in schedule length is shown for different noisy channels. The schedule length reduction was computed as: $|\frac{T_{LP-MLSP} - T_{TDMA}}{T_{TDMA}}|$, where $T_{LP-MLSP}$ is the schedule length obtained by solving LP-MLSP, and T_{TDMA} is the schedule length of a TDMA system ($K = 1$). Clearly, the scenarios where the multiple access channel is better exploited are those of low SNR, or high-noise scenarios. On the other hand, for low-noise scenarios, the schedule length reduction is not very significant, even with a large decoding capability. For example, for $SNR = 100$ dB, the schedule length reduction is at most 10 %, even with a decoding capability of $K = 5$. Thus, a simple TDMA scheme may be preferred for this kind of scenario. For the values of W, P_0, and γ given above, Fig. 7 shows the capacity regions of two transmitters, both located at a distance of 100 m of the receiver node. The figure clarifies the reasons behind the better improvement (by using multiple access capability) in schedule length in high-noise scenarios; for $SNR = -10$ dB, the region is almost rectangular, which implies that scheduling simultaneous transmissions can lead to a significant schedule length reduction. On the other hand, the capacity region becomes more triangular as the SNR increases. Thus, the (multi-access) capacity region approaches that of TDMA, which is bounded by the dashed lines.

Fig. 6(c) shows the schedule length surcharge, which is defined as $\frac{T_{HS} - T_{LP-MLSP}}{T_{LP-MLSP}}$, where T_{HS} is the schedule length produced by HS. This metric quantifies the gap between the optimal schedule length and that of HS. While in a very low-noise scenario, the surcharge is as large as 50% (at $K = 4$), the gap is substantially reduced as the SNR increases; to one extreme, for $SNR = 100$ dB, the schedule length obtained with HS is at most 5% (at $K = 2$) larger than the optimal. Finally, for reference purposes, Fig. 6(d) shows the running time spent by a Pentium Dual Core 2 GHz/3 GB of memory

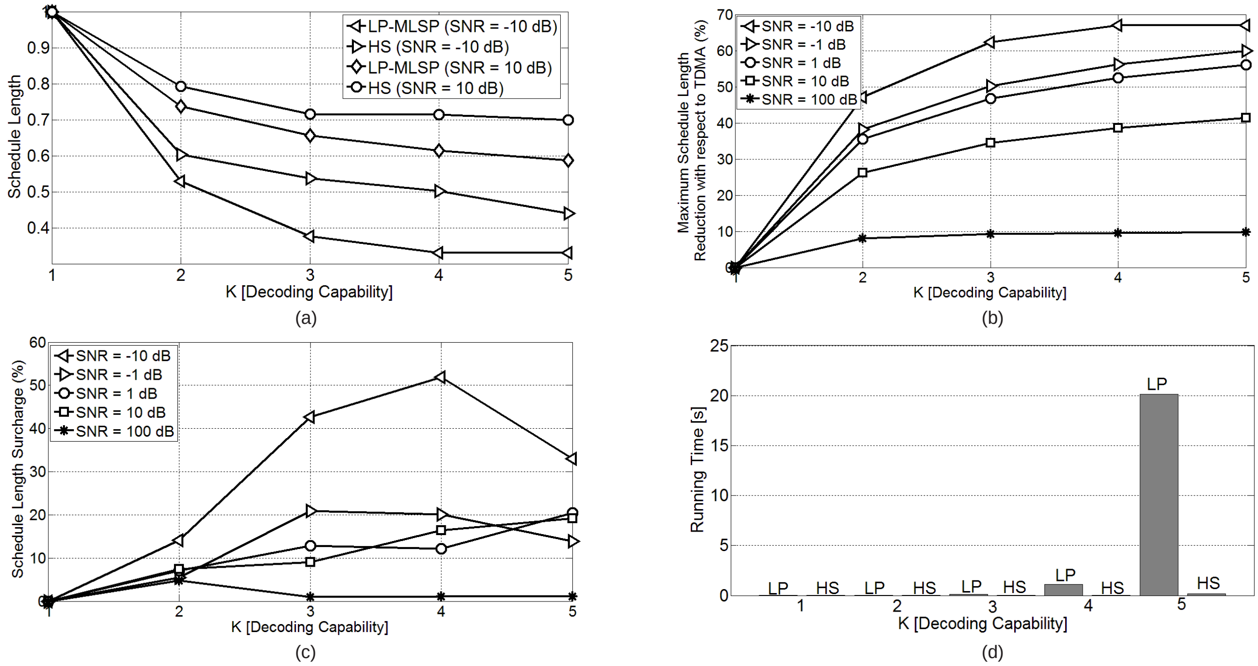

Fig. 6. Performance studies.

to find: i) the optimal solution, given by LP-MLSP (denoted as LP); and ii) the solution provided by HS. The running time of LP increases very fast beyond $K = 3$, since the number of decoding permutations needed to find the optimal solution is lower bounded by $c_1^K, c_1 > 1$ (Eq. (9)). The running time of HS, on the other hand, is mostly flat with K, since its complexity is $O(n^2)$.

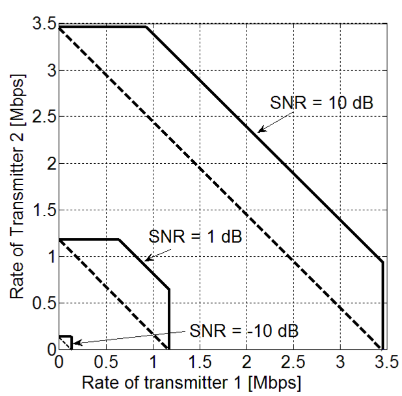

Fig. 7. Capacity regions of two transmitters located at a distance of 100 m of the receiver node.

VI. Conclusion

We have presented an optimization model for the minimum length scheduling problem in single-hop multiple access wireless networks. The model incorporates the capacity region of multiple access channels into scheduling decisions, such that the multiple access capability is fully exploited by making simultaneous transmitters operate at sum-rate. Numerical examples demonstrated that the schedule length can be significantly reduced in low-noise scenarios by increasing the decoding capability of the receiver node. Because of the high-complexity of the model, we have also presented a low-complexity heuristic algorithm, whose performance was shown to be *close* to optimal according to our simulation results; we conjecture that the schedule length produced by the proposed heuristic is factor-two optimal; i.e., its schedule length is at most two times the optimal schedule length. We plan to demonstrate this in our future work, and to explore further for better heuristics and approximation algorithms.

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